Universitat Autònoma de Barcelona

2017/2018

Partial Differential Equations

Code: 42244
ECTS Credits: 6

Degree	Type	Year	Semester
4313136 Modelling for Science and Engineering	OT	0	2

Contact

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Use of languages

Principal working language: english (eng)

Teachers

Angel Calsina Ballesta

Prerequisites

Students should have basic knowledge of calculus, partial differential equations as well as basic skills in programing.

Objectives and Contextualisation

Partial differential equations allow deterministic mathematical formulations of phenomena in physics and engineering as well as biological processes among many other scenarios. The objective of this course is to present the main results in the context of partial differential equations that allow learning about these models and to study numerical methods for the approximation of their solution.

Skills

- Analyse, synthesise, organise and plan projects in the field of study.
- Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
- Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
- Apply techniques for solving mathematical models and their real implementation problems.
- Formulate, analyse and validate mathematical models of practical problems in different fields.
- Isolate the main difficulty in a complex problem from other, less important issues.
- Present study results in English.
- Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
- Use appropriate numerical methods to solve specific problems.

Learning outcomes

1. Analyse, synthesise, organise and plan projects in the field of study.

2. Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
3. Apply partial derivative equation techniques to predict the behaviour of certain phenomena.
4. Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
5. Extract information from partial derivative models in order to interpret reality.
6. Identify real phenomena as models of partial derivative equations.
7. Isolate the main difficulty in a complex problem from other, less important issues.
8. Present study results in English.
9. Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
10. Solve real problems by identifying them appropriately from the perspective of partial derivative equations.
11. Use appropriate numerical methods to study phenomena modelled with partial derivative equations.

Content

Introduction: General classification of partial differential equations, examples of models. Transport equation, method of characteristics.

1. Parabolic equations

Fourier method. Heat equation. Fundamental solution, Gaussian kernel, convolution and solution formula for the pure initial value problem. Maximum principle and uniqueness of the solution.

Numerical Methods: Finite difference methods for scalar parabolic equations: Euler Explicit, Euler Implicit and Crank-Nicolson methods: Von Neumann stability test. Parabolic stability CFL condition. Examples

2. Eliptic equations

Theory: Steady-state problems. Polar/Spherical coordinates: radial solutions. Dirichlet and Neumann boundary value problems. Poisson kernel.

Applications. Euler-Lagrange equations associated to variational problems. Numerics and examples.

3. Hyperbolic equations

Scalar Conservation Laws. Weak solutions. Burgers equation. Shock waves and expansions fans. Hamilton-Jacobi equations and viscosity solutions. Introduction to the Level Set Method. Eikonal equation.

Numerical Methods: Finite difference methods in conservation form. Shock-capturing schemes. Monotone schemes: Lax-Friedrichs and upwind schemes. Convergence and stability conditions. Entropy-satisfying schemes. Examples. Level set method applications.

4. Miscellaneous topics

Introduction to fluid dynamics: Hyperbolic Systems of Conservation Laws: Euler equations for incompressible and compressible fluids. Numerical methods.

Introduction to kinetic theory of gases.

Methodology

Lectures, computer laboratory sessions and individual activities.

Activities

Title	Hours	ECTS	Learning outcomes
Type: Directed			
Computer laboratory sessions	8	0.32	1, 3, 4, 8, 9, 11
Lectures	30	1.2	1, 2, 3, 4, 5, 6, 7, 9, 10
Type: Autonomous			
Completion of assignments	36	1.44	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Preparation	18	0.72	1, 2, 3, 4, 5, 6, 7, 9, 10, 11
Project	50	2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Evaluation

It is necessary to attend at least 80% of all sessions. The evaluation will be accordingly with the following:

Class assignments: 30% of the final qualification will correspond to class assignments proposed along the semester.

Project: 40% of the final qualification will correspond to a project. The project will involve theoretical and numerical development and will be delivered in the form of a written report and oral presentation.

Examn: 30% at the end of the course

Evaluation activities

Title	Weighting	Hours	ECTS	Learning outcomes
Class assignments	30%	2	0.08	2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Exam	30%	3	0.12	2, 3, 4, 5, 6, 8, 10, 11
Project report and project presentation	40%	3	0.12	1, 3, 8, 11

Bibliography

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