

**Algebraic structures**

Code: 100096  
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	2

**Contact**

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**Use of languages**

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: Yes  
Some groups entirely in Spanish: No

**Teachers**

Francesc Perera Domènech  
Joan Claramunt Caros  
Álvaro Sánchez Madrigal

**Prerequisites**

the academic prerequisites consist of the contents of the courses "Fonaments de les Matemàtiques" and "Àlgebra Lineal" from the first year. The ability to manipulate algebraic operations, and the familiarity you already acquired with arithmetic operations and permutation group will be enhanced and you will enter a further layer up in abstraction, a common feature in mathematics.

**Objectives and Contextualisation**

The main objective of this course is to introduce the basic algebraic structures: groups, commutative rings and fields. A lot of emphasis will be put in getting to know a variety of commutative rings, we will study highly non-trivial examples of those and we will study ways to produce even more examples. The sorts of rings we will encounter are typically those that appear in the theory of divisions. In the last part of the course we will introduce the notion of a field and field extensions and we will focus more particularly on finite fields.

**Content**

**1. Groups.**

- Groups, subgroups, homomorphisms.
- Lagrange's Theorem.
- Normal Subgroups. Quotient Groups.
- Isomorphism Theorems.
- Groups acting on sets.

- Singular Groups: cyclic, symmetric and some abelian groups.

## **2. Commutative Rings.**

- Rings, ideals, quotient rings, principal ideals.
- Morphisms. Isomorphism Theorems for rings.
- Maximal and Prime Ideals. Zorn's Lemma.
- Fraction field of a Domain.
- Polynomial Rings.

## **3. Factorization.**

- Irreducibles and primes. Unique Factorization Domains.
- Greatest common divisor.
- Principal Ideal Domains.
- A Principal Ideal Domain is a Unique Factorization Domain.
- Factorization in polynomial rings. Eisenstein's criterion.

## **4. Finite Fields.**

- Fields, characteristic, subfields.
- Existence and unicity of finite fields.