2018/2019

Galois theory

Code: 100102
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	1

Contact

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Use of languages

Principal working language: catalan (cat)

Some groups entirely in English: No

Some groups entirely in Catalan: Yes

Some groups entirely in Spanish: No

Teachers

Francesc Bars Cortina

Prerequisites

Background on Group Theory is required (e.g. the notions introduced and studied in "Estructures Algebraiques"). Groups are used in an essential way in this module. Thus, in order to be able to work with concrete examples, it is particularly interesting to have some familiarity with groups of small order.

It is also important to be familiar with basic notions of Ring Theory (again, the ones introduced in "Estructures Algebraiques). Of particular importance are the notions related to irreducible polynomials, as well as the theory of finite fields.

Objectives and Contextualisation

The main objective of this module is to develop the notions of Galois Theory and their applications to problems related to compass and straighthedge constructions. The latter problems arise as some of the oldest in the History of Mathemathics. Their roots can be traced back to the Babylonia era and culminates brilliantly with the work of Évarist Galois, whose work develops the theory of solvability by radicals.

The modern approach to Galois Theory constitutes a central theme in Algebra, since the abstract methods used show the power of (previously introducted) tools in action. Thus, the translation of a problem to Field Theory, and subsequently to Group Theory (and back) show how abstract, seemingly different branches of of Mathematics interact to solve a classical, more applied problem.

We will start introducing the problem of solving an equation by radicals in its historical context. Next, Field Theory will provide the formal framework where to formulate the problem and study effectively the Galois Theory of equations.

A fundamental tool here is provided by the techniques coming from Group Theory, particularly when it comes to examples and manipulation. However, due to time constraints, we shall review only the most basic concepts and refer to the notions studied in the course "Estructures Algebraiques".

Skills

- Demonstrate a high capacity for abstraction.
- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- For the students to have developed the necessary learning skills to undertake further studies in a highly autonomous manner.
- For the students to know how to apply their knowledge to their work or vocation in a professional manner and possess the skills that tend to be shown by producing and defending arguments and solving problems in their field of study.
- Understand and use mathematical language.

Learning outcomes

1. Calculate groups of low degree Galois equations and deduce their resolvability by radicals.
2. For the students to have developed the necessary learning skills to undertake further studies in a highly autonomous manner.
3. For the students to know how to apply their knowledge to their work or vocation in a professional manner and possess the skills that tend to be shown by producing and defending arguments and solving problems in their field of study.
4. Manipulate expressions involving algebraic and transcendent elements.
5. Relate geometric constructions with algebraic extensions.

Content

1. Basic Notions

Polynomial equations: the formulas for small degree.

Rings, ring homomorphisms. The field of fractions of a commutative domain.

The characteristic of a field.

2. Field extensions

Algebraic and transcendental elements.

The degree of a field extension. The multiplicativity formula for degrees.

Algebraic extensions.

Extensions de homomorphisms. The group Gal(L/K).

The spitting field of a polynomial.

3. Normal and separable extensions

Normal extensions.

Formal derivative of a polynomial and polynomials of multiple roots.

Separable elements and separable extensions.

Finite fields.

4. The Fundamental Theorem of finite Galois Theory

Galois extensions. Artin's Theorem.
Galois correspondence: El fundamental theorem.

5. Galois theory of equations.

Solvability by radicals and solvable groups.

Cyclotomic and cyclic extensions.

Galois Solvability Theorem .

Polynomials whose Galois group is S_p, where p is prime.

6. A particular example: Constructions with compass and straightedge.

Methodology

There will be two lectures and one tutorial per week, during 15 weeks. In addition, there will be 3 seminar sessions of 2 hours each, distributed in the semester. Students are strongly encouraged to attend lectures, tutorials, and seminars.

During the lectures, the main tools needed for understanding the subjet and also for problem solving will be introduced. Problem solving will be the main focus in the tutorials, where also a better understanding of the concepts introduced in the lectures will be achieved. Students participation in the form of discussion will be part of the methodology.

In seminars, students participation will be more prominent as these are designed in the form of hands-on exercises and focusing, in particular, in manipulation of examples.

Various resources will be offered through moodle. In particular, problems/seminars and additional material that may complement the subject matter of the course.

Activities

Title	Hours	ECTS	Learning outcomes
Type: Directed			
Lectures	30	1.2	1, 4, 2, 3, 5
Seminars	6	0.24	1, 4, 2, 3, 5
Tutorials	15	0.6	1, 4, 2, 3, 5
Type: Autonomous			
Course work (from lectures)	27	1.08	1, 4, 2, 3, 5
Exams preparation	16	0.64	1, 4, 2, 3, 5
Problem solving	40	1.6	1, 4, 2, 3, 5
Seminar preparation	10	0.4	1, 4, 2, 3, 5

Evaluation

The distribution of marks will be done as follows:

35% of the grade corresponds to the Intersemester exam.

15% of the grade corresponds to problem and/or practical assignments.

50% of the grade corresponds to the exam at the end of semester.

There will be a resit exam, that will substitute the grade corresponding to the Intersemester exam and the exam taken at the end of semester.

Qualification of Non-Assessed. A student will be classified as non-assessed if the following conditions are met:
He/she does not turn up to the exam realized in January-February,
he/she does not turn up to the resit exam in February.

Higher academic distinction (Honors): After the exam carried out at the end of the semester, the higher academic distinctions will be awarded. In case the maximum number of distinctions has not been reached, the possibility of awarding the remaining ones after the resit exam will be considered. The latter exam can be used for students to improve their final grade in any case.

Evaluation activities

Title	Weighting	Hours	ECTS	Learning outcomes
Exam	50%	3	0.12	1, 4, 2, 3, 5
Intersemester exam	35%	2	0.08	1, 4, 2, 3, 5
Problem assignments	15%	1	0.04	1, 4, 2, 3, 5

Bibliography

F.Bars, Teoria de Galois en 30 hores, http://mat.uab.cat/~francesc/docencia2.html

David A. Cox, Galois Theory. Hoboken : Wiley-Interscience, cop. 2004
http://syndetics.com/index.aspx?isbn=0471434191/summary.html&client=autbaru&type=rn12

D. S. Dummit, M. R. Foote, Abstract Algebra, Wiley, 2004.

D.J.H. Garling. A course in Galois Theory. Cambridge Univ. Press, 1986.

J. Milne. Fields and Galois Theory, http://www.jmilne.org/math/

P. Morandi. Fields and Galois Theory. GTM 167, Springer.

S. Roman. Field Theory. GTM 158, Springer.

Ian Steward "Galois Theory" Chapman & Hall / CRC, 2004
http://syndetics.com/index.aspx?isbn=1584883936/summary.html&client=autbaru&type=rn12

Additiona bibliography:

Michael Artin, "Algebra" Prentice Hall, cop. 2011
http://syndetics.com/index.aspx?isbn=9780132413770/summary.html&client=autbaru&type=rn12

T. Hungerford, "Algebra" New York : Springer-Verlag, cop. 1974
http://syndetics.com/index.aspx?isbn=0387905189/summary.html&client=autbaru&type=rn12

Jean-Perre Tignol, "Galois' Theory of Algebraic Equations". World Scientific 2001

A. M. de Viola Priori, J.E. Viola-Priori. Teoría de cuerpos y Teoría de Galois. Reverté (2006).

Galois' life (novel, in Catalan):

Josep Pla i Carrera. Damunt les espatlles de gegants. 1ra Edició: Editorial la Magrana. 2na Edició: Edicions FME http://www.fme.upc.edu/ca/arxius/damunt-les-espatlles-dels-gegants_jpla

Some interesting links:

http://www.galois-group.net
http://godel.ph.utexas.edu/~tonyr/galois.html
htpp://www-groups.dcs.st-andrews.ac.uk/%7Ehistory/Mathematicians/Galois.html
Curiosinats origami: Robert J. Lang: http://www.langorigami.com
Tom Hull: http://www.merrimack.edu/thull/~omfiles/geoconst.html
Koshiro Hatori: http://origami.ousaan.com/library/conste.html
http://www.langorigami.com/science/mathlinks/mathlinks.php4.