2018/2019

Topology of manifolds

Code: 100114
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

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Use of languages

Principal working language: spanish (spa)
Some groups entirely in English: No
Some groups entirely in Catalan: No
Some groups entirely in Spanish: Yes

Teachers

Juan Luis Durán Batalla

Prerequisites

It is better to have succeded in the course "Diferential Calculus".

Objectives and Contextualisation

Ever since the concept of homeomorphism was clearly defined, the "ultimate" problem in topology has been to classify topological spaces "up to homeomorphism". That this was a hopeless undertaking was very soon apparent, the subspaces of the plane R2 being an obvious example. From this impossibility were born algebraic and differential topology, by a shift of emphasis which consisted in associating "invariant" objects to some types of spaces (objects are the same for two homeomorphic spaces). At first these objects were integers, but it was soon realized that much more information could be extracted from invariant algebraic structures such as groups and rings.

(Jean Dieudonné, A history of algebraic and differential topology 1900--1960)

If topology is concerend with the classification of shapes, in this course we will introduce some of the most basic algebraic tools used in this classification: de Rahm cohomology and the fundamental group. In particular de Rahm cohomology, a natural genralization of differential calculus, attaches to each manifold a series of finite dimensional vector spaces which encode various features of a manifold: its dimension, its orientability, higher orientability properties (spin structures etc.). The objective of this course is to construct these vector spaces and present some of the tools used to extract from them relevant information.

Content

We will cover the following topics:

- Smooth manifolds.
- Smooth atlas, change of charts.
- Tangent and cotangent bundles
- Vector fields

- De Rahm complex
- Differential forms
- De Rahm complex
- Cohomology
- Euler characteristic
- Poincaré duality

- Fundamental groups and covering spaces
- Homotopy
- Covering space
- Universal covering space
- classification of covering spaces

As an application we will prove some of these results:

- Classification of closed surfaces
- Brouwers fixed point theorem
- Jordan-Brouwer separation theorem
- Topological invariance of dimension.