Universitat Autònoma de Barcelona

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Dynamic Systems

Code: 100118
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

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Use of languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Prerequisites

Ordinary differential equations: existence and uniqueness of solutions of the Cauchy problem.

Linear differential systems with constant coefficients.

Linear algebra: spaces and vector subspecies, diagonalization.

Objectives and Contextualisation

This course is an introduction to the modern theory of dynamic systems. A first objective is to familiarize the student with the notion of a dynamical system and the basic concepts of this theory: stability, attractor, invariant sets, alpha and omega limits, etc. The second objective is to understand how is the local behavior, in discrete and continuous dynamical systems, near an equilibrium point or a periodic orbit. This local behavior is based on the topological classification of linear systems in R^n, both those that are determined by the flow of ordinary differential equations (continuous dynamical systems) and those that come from the iteration of functions (discrete dynamical systems). Linear systems are very important because they are the first approach of more complicated systems.

The qualitative theory of differential equations began with the work of Poincaré towards the year 1880 in relation to his works of Celestial Mechanics. The main idea is to know properties of the solutions without needing to solve the equations. This qualitative approach, when combined with the right numerical methods, is, in some cases, equivalent to having the solutions of the equation. We will present the basic results of the qualitative theory (Liapunov theorems, Hartman theorem and theorems of the stable and central varieties) on the local structure of equilibrum points and periodic orbits. Additionaly, in R^2 Begin in the problem of detecting the existence of periodic orbits via the Poincaré-Bendixson and Bendixson-Dulac theorems.

Finally, we introduce the techniques to study discrete global dynamics. The main example will be the unimodal maps. They (for some parameter values) present a dynamic that simply leads to the notion of chaotic system. For these systems the numerical approach is not feasible and to understand its dynamics new tools are needed. Chaotic systems are often presented in applications (problems of weather forecasting, electrical circuits, etc.).

Content

1. Dynamical systems in Euclidean spaces.

- Dynamical systems defined by differential equations and by difeomorphisms.
- Orbits; Critical points and periodic orbits.
- Invariant sets and limit sets.
- Attractors. Liapunov stability.
- Conjugation of dynamic systems.

2. Study of local dynamics, discrete and continuous.

- Phase portraits near equilibrim and regular points.
- Topological classification of continuous and discrete linear systems.
- Stability (Functions of Liapunov)
- Hartman theorems, of the stable variety and of the central variety.
- Periodic orbits: Application of Poincaré and stability.

3. Global dynamics in continuous systems.

- Ordinary differential equations in R2 (Theorem of Poincaré-Bendixon, Theorem of Bendixon-Dulac, Existence and unicity of limit cycles, ...)
- Ordinary differential equations in dimension greater than 2.

4. Global dynamics in discrete systems.

- Iteration in dimensions 1 and 2.
- Unimodal applications.
- Chaos Bernoulli's shift. Smale's Horseshoe.