Universitat Autònoma de Barcelona

2019/2020

Calculus of several variables and optimization

Code: 100093
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	1

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Josep Maria Burgués Badía
Joan Josep Carmona Domènech
Julià Cufí Sobregrau
José González Llorente
Georgios Sakellaris

Prerequisites

The main prerequisite is a standard first-year course in Calculus, covering differential and integral calculus in one real variable. If the student has not passed such a first year course, it is required that, at least, the student understands the notion of convergence of functions or sequences, as well as those of continuity, derivability and integrability of functions. It is also very important that the student is familiar with the computation of limits, differentiation rules, the fundamental theorem of calculus and Taylor's expansion of elementary functions.

Objectives and Contextualisation

The main objective of this course is to get familiarized with the basic techniques of differential and integral calculus in several variables and the most important concepts of vector analysis.

The first part of the course covers basic topics related to the Euclidean space and its metric and topological structure. Next, the key concept is that of differential as a linear approximation of a differentiable function and other better approximations, in terms of higher order differentials, and how the behavior of these approximations translates into local properties of the function. As in the case of one variable, the techniques of the course will be applied to the resolution of different problems (geometric problems and optimization). One then proceeds to study curves and regular surfaces, tangential planes and local coordinates.
The second part of the course is devoted to multiple integrals and vector analysis, covering topics such as change of order of integration, change of variables, Green and Stokes's theorems.

Competences

- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Know how to apply the theorems of Inverse Function and of the implicit function to specific problems.
2. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
3. Understand the basic results of Differential Calculus in different real variables.

Content

1. Differential calculus in several variables:

- Basic geometrical and topological notions in the Euclidean space. Limits and continuity. Graphics and level sets

- Differentiability. Basic properties. Partial derivatives and directional derivatives

- Higher order differentials. Taylor's Formula. Relative extremes and analysis of critical points.

- The inverse function theorem. Changes of coordinates.

- The implicit function theorem. Geometric viewpoint, regular submanifolds.

- Constrained extrema. Lagrange multipliers

2. Integration

- The Riemann integral of bounded functions on rectangles. Basic properties

- Fubini's theorem.

- Integration on more general sets

- Change of variables and the Jacobian.

- Length and area. Integration on curves and surfaces.

3. Vector analysis

- Orientable surfaces.

- Circulation and flow of a vector field.

- Divergence and rotation of a vector field. Conservative and solenoidal fields.

- Theorems of Green, Gauss and Stokes.

Methodology

There are three type of activities the student is supposed to attend: the lectures (3 hours /week) mainly concerned with the description of the theoretical concepts, problem solving sessions (1 hour/week) and seminars (2 hours on alternate weeks), similar to the problem solving sessions but where students work in groups supervised by a teaching assistant.

The course has a web page in the UAB online campus gathering all information and communications between students and professors, and where all material, including problem sheets, solutions, etc are published regularly.

Students are supposed to submit a couple of exercise sets to be evaluated in a personalized interview.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	39	1.56	
Problem session	15	0.6	
Working seminars	13	0.52	
Type: Autonomous			
Solving problems	95	3.8	
Studying theoretical concepts	53	2.12	

Assessment

A continuous assessment is done consisting in a mid-term test (P) and two submissions of exercise sets (LL1,LL2), mandatory and evaluated in a personalized interview. At the end there is a final exam (F) and a resit exam (R)

The final score is obtained in two steps. Let P,LL,F,R denote respectively the scores (between 0 and 10) of the mid-term test, the mean of the two exercise submissions, the final and the resit exams.

With P,F we compute NE=max(F; (0,6)F+(0,4)P) and then C1=(0,80)NE+(0,20)LL. If C1 is greater or equal to 5, C1 is the final score. Otherwise, the students that have submitted the two exercise sets may attend the resit exam. Then the final score is C2=(0,80) R+(0,20)LL.

Students with C1 greater or equal to 5 may attend R to improve their grades, in which case the final score is (C1+C2)/2.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes

Final test	40%	3	0.12	3, 2, 1
Graded seminar	15%	2	0.08	3, 2, 1
Graded seminar	15%	2	0.08	3, 2, 1
Mid-term test	30%	3	0.12	3, 2

Bibliography

- Cálculo Vectorial, J.E. Marsden y A.J.Tromba, Addison Wesley Longman. It is recommended to solve some of the many problems at the end of each section. Is a book that will be always useful and that we will follow closely.
- Functions of several variables, Martin Moskowitz and Fotios Paliogiannis, World Scientific, 2011. It is a book that adapts very well to the course and that we will closely follow.
- Analysis II, Terence Tao, Hindustan Book Agency, Texts and readings in Mathematics 38, 2006. The author was a Field Medal recipient in 2006. It is thus interesting to see how he presents the material. There are wo concise chapters on Lebesgue measure and integral.
- Functions of several variables, Wendell Fleming, Undergraduate texts in Math, Springer. It is a classic a bit above our level. It might be useful to complement our course, to see complete proofs and a more theoretical perspective than ours.
- Second year calculus, David Bressoud, Undergraduate texts in Math, Springer, 1991. It is an original presentation of vector calculus based on physics. The subtitol is: "From celestial mechanics to special relativity".
- Lecture Notes of the course, by Joan Verdera and J.Bruna.