Universitat Autònoma de Barcelona

2019/2020

Mathematical analysis

Code: 100094
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	1

Contact

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Use of Languages

Principal working language: catalan (cat)

Some groups entirely in English: No

Some groups entirely in Catalan: Yes

Some groups entirely in Spanish: No

Prerequisites

In order to be able, for a student, to follow the course, it is very important that the student has succeeded in the first course subject Funcions de Variable Real (functions of one real variable). If this is not the case, it is essential that, the student understands the notions of convergence of sequences and continuity, differentiability and integrability of functions. It is also crucial that the student has enough mathematical skills in the manipulation of limits, Taylor series representation of functions...

Objectives and Contextualisation

For a student to succeed in this subject is is essential to acquire the following capacities.

Theoretical skills.

1. Understand the notion of series convergence and improper integrals.

2. Know about the most important criteria to decide the convergence of series and improper integrals.

3.Fully understand the notion of uniform convergence of a sequence of functions.

4. Understand the results that relate the uniform convergence on one side, and the notions of continuity, derivability and integrability on the other.

5. Understand why it is important to consider power series in the complex context.

6. Understand the results that involve the regularity of functions defined from integrals depending on a parameter.

7. Know about the principal results that relate the regularity of a function and the convergence of a Fourier series.

8. Understand and be able to reproduce the proofs of the main results of the subject.

Problem solving skills

1. Be able to apply the different criteria to decide whether a series or an improper integral converge.

2. Be able to compute the radius of convergence of a power series and know how to sum them in some concrete situations.

3. Be able to represent a function as an infinite sum of terms, as a power series, if possible.

4. Prove results involving uniform convergence of sequences of functions.

5. Be able to compute the Fourier coefficients of functions and be able to compute the sum of some complex series applying the Fourier series results.

6. Be able to relate the different main results of the subject and apply them to solve concrete problems.

Competences

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Demonstrate a high capacity for abstraction.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.

Learning Outcomes

1. Calculate integrals of functions of a variable.
2. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
3. Understand the concepts of series and integral convergence and master the most important criteria of convergence.
4. Understand the relationship between uniform convergence and continuity, derivability and integrability of the functions of a variable.

Content

1. Series of numbers.

1.1 Extension of the notion of limit of a sequence.
1.2 Notion of convergent series.
1.3 Non-negative series. Convergence criteria.
1.4 Absolute and condicional convergence.
1.5 Leibniz, Dirichlet and Abel criteria.
1.6 Rearranging series. The Riemann series theorem.

2. Univorm convergence and power series.
2.1 Sequences of functions.
2.2 Pointwise and uniform convergence.
2.3 Uniform convergence and continuity, differentiability and integrability.
2.4 Function series.
2.5 Weierstrass M test.
2.6 Existence of continuous functions nowhere differentiable.
2.7 Power series and radius of convergence.
2.8 Abel Theorem.
2.9 Analytic functions.
2.10 Approximation of continuous functions by polynomials: Weierstrass theorem.

4. Improper Integrals.
4.1 Extension of the notion of Riemann integral for non-bounded functions or intervals.
4.2 Convergence of improper integrals.
4.3 Convergence criteria for positive functions.
4.5 Continuity and derivability for functions with more than one variable.
4.6 Integrals depending on one parameter.
4.7 The Euler Gamma function. Stirling's theorem.

5. Fourier series.
5.1 L^2 functions.
5.2 Trigonometric polynomials. Fourier coeficients. Fourier series.
5.3 Pointwise and uniform convergence of a Fourier series.

5.4 Gibbs phenomena.
5.5 Parseval's identity.

Methodology

It is just explained above.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Final Exams	4	0.16	
Partial exams	2	0.08	
Problem sessions	14	0.56	
Seminar sessions	14	0.56	
Theory sessions	42	1.68	
Type: Supervised			
Doubt clearing sessions student-professor	4	0.16	
Type: Autonomous			
At home work	46	1.84	
Exam preparation	30	1.2	
Preparation	4	0.16	
Solve problems and exercises	60	2.4	

Assessment

Therefore, there will be a mark E of seminars, a partial exam with a P note and a final exam with a grade F. If 0.2E + 0.3P + 0.5F is greater than or equal to 5, the student pass the subject.

Otherwise there will be a voluntary second-chance / improvement exam. In case of doing this improvement exam, the grade will be computed in the following way: 0.2E + max (0.3P + 0.5F, 0.8R),

where R is the grade obtained in this last second-chance exam.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final Exam	50%	3	0.12	1, 4, 3, 2
First partial exam	30%	1	0.04	4, 2
Practice sessions with mark	20%	1	0.04	1, 4, 3, 2

Bibliography

1. J. Casasayas i Mª C. Cascante. *Problemas de análisis matemático.* Edunsa Ediciones y Distribuciones Universitarias s.a., Barcelona, 1990.

2. F. Galindo i altres. *Guía Práctica de Cálculo Infinitesimal en una variable real.* Ed. Thomson, Madrid 2003.

3. J. M. Ortega. *Introducció a l'Anàlisi Matemàtica*. Manuals de la Universitat Autònoma de Barcelona 4, Bellaterra 1990.

4. C. Perelló. *Càlcul Infinitesimal: amb mètodes i aplicacions*. Enciclopèdia Catalana, Barcelona, 1994.

5. W. Rudin. *Principios de Análisis Matemático*. McGraw-Hill, Mèxic, 1981.

6. G. P. Tolstov. *Fourier Series,* Edover Publications, New York, 1976.