

**Algebraic structures**

Code: 100096  
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	2

**Contact**

Name: Jaume Moncasi Solsona  
Email: Jaume.Moncasi@uab.cat

**Use of Languages**

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: Yes  
Some groups entirely in Spanish: No

**Teachers**

Francesc Xavier Xarles Ribas  
Laurent Cantier

**Prerequisites**

the academic prerequisites consist of the contents of the courses "Fonaments de les Matemàtiques" and "Àlgebra Lineal" from the first year. The ability to manipulate algebraic operations, and the familiarity you already acquired with arithmetic operations and permutation group will be enhanced and you will enter a further layer up in abstraction, a common feature in mathematics.

**Objectives and Contextualisation**

The main objective of this course is to introduce the basic algebraic structures: groups, commutative rings and fields. A lot of emphasis will be put in getting to know a variety of commutative rings, we will study highly non-trivial examples of those and we will study ways to produce even more examples. The sorts of rings we will encounter are typically those that appear in the theory of divisibility. In the last part of the course we will introduce the notion of a field and field extensions and we will focus more particularly on finite fields.

**Competences**

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Work in teams.

## Learning Outcomes

1. Calculate the maximum common divisor and factorisation of whole numbers and polynomials.
2. Construct quotient groups and rings and finite bodies and operate within them.
3. Operate in some simple groups (such as cyclic, dihedral, symmetric and abelian).
4. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
5. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
6. Work in teams

## Content

### 1. Groups.

- Groups, subgroups, homomorphisms.
- Lagrange's Theorem.
- Normal Subgroups. Quotient Groups.
- Isomorphism Theorems.
- Groups acting on sets.
- Singular Groups: cyclic, symmetric and some abelian groups.

### 2. Commutative Rings.

- Rings, ideals, quotient rings, principal ideals.
- Morphisms. Isomorphism Theorems for rings.
- Maximal and Prime Ideals. Zorn's Lemma.
- Fraction field of a Domain.
- Polynomial Rings.

### 3. Factorization.

- Irreducibles and primes. Unique Factorization Domains.
- Greatest common divisor.
- Principal Ideal Domains.
- A Principal Ideal Domain is a Unique Factorization Domain.
- Factorization in polynomial rings. Eisenstein's criterion.

### 4. Finite Fields.

- Fields, characteristic, subfields.
- Existence and unicity of finite fields.

## Methodology

This subject has three hours per week of theory classes, one hour per week of problem classes, and, during the semester, eight seminar sessions, two hours each.

Students will have the lists of problems previously to be able to work before the problem classes. In classe, you can not solve all the problems but we recommend that students work on their own and ask the teachers their questions. In the seminar sessions the students will work under the supervision of the teacher. In some of these seminars, some exercises will be given that will count for the final mark of the subject.

## Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Directed	16	0.64	
Theory classes	43	1.72	
Type: Supervised			
Seminars	14	0.56	
Type: Autonomous			
Seminar preparation	145	5.8	

## Assessment

- The student will obtain 10% of the mark of the subject with the delivery of exercises previously done. We denote by LP the mark on 10 calculated as the average of deliveries.

-A written examination will be carried out to evaluate the theoretical and practical knowledge of the subject in mid-semester. The mark on 10 (P1) of this examination will count 30% of the mark of the course.

-In some of the seminars, classroom exercises will be given. They will be short exercises to evaluate practical aspects of the newly completed seminar. The mark (S) on 10, calculated as the average of the seminar marks, will count 10% of the marks of the subject.

-50% of the mark of the subject will correspond to the mark P2 obtained in the final examination. This examination will evaluate the student's practical and theoretical knowledge.

The mark of the subject is obtained by the expression  $N = 0,10.LP + 0,10.S + 0,30.P1 + 0,50.P2$ . The student will pass if N is greater or equal than 5.

The qualification of Excellent with honours will be awarded based on N

There will be a resit examination corresponding the final examination. Only students with the mark N less than 5 and who have been presented to the examination that give rise to the P1 and P2 marks may attend the resit examination. In this case, the final mark of the subject will be calculated as  $\text{Max}(N; 0,10.LP + 0,10.S + 0,30.P1 + 0,50.R)$  where R denotes the mark of examination resit

## Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Autonomous	145 hours , 5,8 ECTs	4	0.16	1, 2, 3, 5, 4, 6
Supervised	14 hours, 0,56 ECTS	3	0.12	1, 2, 3, 5, 4, 6

## Bibliography

- [1] R. Antoine, R. Camps, J. Moncasi. Introducció a l'àlgebra abstracta. Manuals de la UAB, Servei de Publicacions de la UAB, nº 46, Bellaterra, 2007.
- [2] F. Cedó, V. Gisin, Àlgebra bàsica, Manuals de la UAB, Servei de Publicacions de la UAB, nº 21, 2007.
- [3] P. M. Cohn, Algebra, vols. 1 i 2, John Wiley and Sons, 1989.
- [4] J. Dorronsoro, E. Henández, Números, Grupos y anillos, Addison-Wesley, 1996.
- [5] F. Delgado, C. Fuertes, S. Xambó, Introducción al Álgebra: anillos, factorización y teoría de cuerpos, Universidad de Valladolid, 1998.
- [6] J. B. Fraleigh, A First course in Abstract algebra, Addison-Wesley, 1982.
- [7] T. W. Hungerford, Álgebra, Springer-Verlag, 1974.