Universitat Autònoma de Barcelona

2019/2020

Differencial geometry

Code: 100107
ECTS Credits: 12

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	2

Contact

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Use of Languages

Principal working language: catalan (cat)

Some groups entirely in English: No

Some groups entirely in Catalan: Yes

Some groups entirely in Spanish: No

Teachers

Eduardo Gallego Gómez

Agustí Reventós Tarrida

Gil Solanes Farrés

Prerequisites

To assimilate the contents of the subject, it is convenient to have a previous knowledge of calculus in several variables (derivation, integration, implicit function theorem), differential equations (theorem of existence and uniqueness of solutions), algebra and linear geometry (diagonalization of self-adjoint endomorphisms, quadratic forms) and topology (homeomorphism, index of a flat curve, Euler characteristic).

Objectives and Contextualisation

The concepts and notions of differential geometry and vector calculus are basic for understanding the physical reality that surrounds us. Their technical applications are also important in the field of engineering, where the objects of study can be represented geometrically by non-linear elements of the three-dimensional space, that is, basically by curves and surfaces.

The main objective is to study the geometric notions that allow theoretical characterization of the shape of these elements (curvature and torsion in the case of a curve, first and second fundamental form in the case of a surface), as well as to develop methods to calculate their metric characteristics (length, area, etc.). It is also important to relate the invariants associated with a curve contained on a surface with the notions and magnitudes of the latter. All these topics will be dealt with in the first two blocks of the course.

In the third block, the classic notions of the vector calculus will be introduced: vector fields and their line and surface integrals, as well as the integral theorems of Green, Gauss and Stokes that relate them. These results will be obtained as particular cases of the Stokes theorem for differential forms. Numerous applications of these theorems will be presented, both in physics and geometry.

Competences

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Recognise the presence of Mathematics in other disciplines.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Apply the integrals of line and surface to recognise some global properties of crooked and surfaces.
2. Know pose and resolve curvilinear and integral integrals of surface.
3. Recognise the nature of the points of an R3 curve Calculate curvature and torsion.
4. Recognise the nature of the points of an R3 surface. Calculate Gauss curvature, mean curvature and principal curvatures.
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
7. Topologically recognise compact spaces and their classification.
8. Understand the applications of vector calculus and differential geometry calculus to physics problems.
9. Use some kind of scientific program to make computations and display surfaces.

Content

1. Curves
Parameterization and length.
Curvature and torsion. Frenet Formulas
Osculating sphere.

2. Surfaces.
Regular surfaces.
First fundamental form.
The Gauss map and the second fundamental form.
Normal curvature and geodesic curvature.
Intrinsic geometry of surfaces. Gauss Egregium theorem.
Geodesics
The default theorem.

3. Vector calculus.
Differential forms.
Varieties and varieties with boundary.
Integration
Stokes theorem and applications.
The Gauss-Bonnet theorem.

Methodology

The course consists of:

1. Theory classes (three hours per week) where the fundamental concepts, illustrated with abundant examples, will be introduced, and the program's themes will be explained. Students will be encouraged to actively participate in class.
2. Problem solving classes (two hours per week) where significant exercises will be solved and certain questions that clarify the notions introduced into the theory classes will be analyzed in detail. The heuristic methods of solving mathematical problems that are characteristic of the subject will be emphasized.
3. Seminars (a session of two hours per week) where the students, under the guidance of the professor, will develop and deepen certain topics that complement the subjects explained in the theoretical classes.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem solving classes	30	1.2	1, 8, 6, 5, 3, 4, 7, 2, 9
Theory classes	45	1.8	1, 8, 6, 5, 3, 4, 7, 2, 9
Tutoring	15	0.6	1, 8, 6, 5, 3, 4, 7, 2, 9
Type: Supervised			
Seminars	28	1.12	1, 8, 6, 5, 3, 4, 7, 2, 9
Type: Autonomous			
Autonomous study	160	6.4	1, 8, 6, 5, 3, 4, 7, 2, 9

Assessment

The mark of the continuous evaluation of the subject, AC, will be obtained from:

1. the marks of two partial exams, E1 and E2,
2. the mark of the delivery of exercises (from the classes of problems or seminars), P,

according to the formula: AC = 0.35 E1 + 0.45 E2 + 0.20 P.

The student passes the subject if AC is greater than or equal to 5. Otherwise, the student has a recovery examination giving rise to a mark ER. The recovery examination can be used to improve the grade but notice that, in all the cases, ER will replace the sum of the two partial examination marks, E1 + E2 . The mark P of delivery of exercises is NOT recoverable and therefore the mark ER of the recovery examination will have a weight of 80% in the final grade. To be able to attend the recovery exam, the student must have been previously evaluated of continuous assessment activities that are equivalent to 2/3 of the total.

It is considered that the student presents himself for the evaluation of the course if he has participated in evaluation activities that exceed 50% of the total.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Delivery of solved problems P	20%	10	0.4	1, 8, 6, 5, 3, 4, 7, 2, 9

Exam E1	35%	4	0.16	1, 8, 6, 5, 3, 4, 7, 2
Exam E2	45%	4	0.16	1, 8, 6, 5, 3, 4, 7, 2
Recovery exam ER	80%	4	0.16	1, 8, 6, 5, 3, 4, 7, 2

Bibliography

1. Manfredo P. do Carmo. Geometría diferencial de curvas y superfícies. Alianza Editorial. 1990
2. Joan Girbau. Geometria diferencial i relativitat. Manuals de la UAB. 1993
3. Michael Spivak. Cálculo en Variedades. Ed. Reverté. 1970
4. Sebastián Montiel y Antonio Ros. Curvas y superfícies. Proyecto Sur. 1998