

Stochastics processes

Code: 100116
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Prerequisites

As a general requirement, in order to be able to follow this subject, good knowledge is required at the practical level of analysis and calculus or, more specifically, integration and series. As more specific requirements, it is necessary to have previously studied a course in Probability and Stochastic Modeling.

Objectives and Contextualisation

The aim of this subject is, on the one hand, to introduce the student in the part of the theory of probability called theory of stochastic processes, whose purpose is to study the random phenomena that evolve over time or in the space. We will see the basic generalities of these models and we will study some specific models.

The discrete Markov chains will be studied in general and in the particular case of the random walk. We will also study the continuous-time Markov chains, such as the Poisson process or the birth and death processes. Finally we will also introduce the Brownian motion.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Effectively use bibliographies and electronic resources to obtain information.
- Generate innovative and competitive proposals for research and professional activities.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.

2. Devise demonstrations of mathematical results in the field of probability and statistics.
3. Effectively use bibliographies and electronic resources to obtain information.
4. Generate innovative and competitive proposals for research and professional activities.
5. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
6. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Discrete-time Markov chains.
 - 1.1. Motivation: the random walk.
 - 1.2. Definitions. Basic properties. Transition matrix.
 - 1.3. Stopping time. Strong Markov property.
 - 1.4. Recurrence and transience.
 - 1.5. Asymptotic behavior. Invariant distribution
 - 1.6. Ergodic theorem
 - 1.7. More aspects about random walk
2. Continuous-time Markov chains.
 - 2.1. Motivation: the Poisson process.
 - 2.2. Basic properties. Generating matrix. Recurrence and transience.
 - 2.3. Invariant distribution
 - 2.4. Ergodic theorem
 - 2.5. More aspects of the Poisson process.
3. The Brownian motion.

Methodology

This subject is semiannual and consists of two hours of theory and one hour of problems per week. There will also be three two-hour seminars.

In theory classes, the teacher plays the main role and we will work with the structure definition-theorem-proof-application.

In the problem classes students will leave on the board to solve the problems. Problem solving will be monitored at all times by the teacher.

On the other hand, in the seminar sessions, the student will work, under the tutelage of the teacher, some practical situations that are related to what has been studied in the theory classes.

These sessions will also allow, students and teachers, to be aware of the evolution in the achievement of the concepts and methods that are introduced in theory classes.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem classes	14	0.56	1, 4, 2, 6, 5, 3
Theory classes	28	1.12	1, 2, 6, 3

Type: Supervised

Seminars	6	0.24	1, 4, 2, 6, 5
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Type: Autonomous

Prepare exams	20	0.8	2, 6, 3
Study of the theory and resolution of problems	64	2.56	1, 4, 2, 6, 5, 3

Assessment

During the semester there will be two partial exams. The first one will take place approximately in the middle of the course and the second will be done at the end of the course. The final mark of the exams, y , will be obtained by doing the arithmetic mean of the qualifications of the two partials.

Throughout the course, problems, tasks and kahoots will be proposed that students can voluntarily resolve and return them corrected. With these problems and volunteer tasks students can obtain a note x .

The final grade of the subject will be obtained by applying the following formula:

$$N(x, y) = x + (1 - 0.1 \cdot x) \cdot y$$

We refer you to the article *Matemáticas y evaluación*, *Materials Matemàtics* volume 2011, by X. Bardina and E. Liz, where the formula is explained in detail.

If necessary, a recovery exam will be scheduled.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	$(1 - 0.1 \cdot x) \cdot 5\% (< 50\%)$	4	0.16	1, 2, 6, 5
Problems and voluntary tasks	$10 \cdot x\%$	6	0.24	1, 4, 2, 6, 5, 3
Recovery exam	100%	4	0.16	1, 4, 2, 6, 5, 3
Second partial exam	$(1 - 0.1 \cdot x) \cdot 5\% (< 50\%)$	4	0.16	1, 2, 6, 5

Bibliography

1. Bardina, X. & Ferrante, M. An excursion into Markov chains. Springer, 2020.
2. Breiman, L. Probability and Stochastic Processes: With a View Toward Applications. Houghton Mifflin Company Boston, 1969.
3. Brémaud, P. Markov Chains: Gibbs measures, Montecarlo simulation, and queues. Texts in Applied Mathematics. Springer, 1998.
4. Feller, W. Introducción a la teoría de probabilidades y sus aplicaciones, Vol I. John Wiley & Sons, 1988.
5. Karlin, S. & Taylor, M.H. A First Course in Stochastic Processes. Academic Press, New York, 1975.
6. Karlin, S. & Taylor, M.H. A Second Course in Stochastic Processes. Academic Press, New York, 1981.
7. Lawler, G.F. Introduction to Stochastic Processes. Chapman and Hall/CRC Probability Series, 1995.

8. Hoel, P.G., Port, S.C. & Stone, C.J. Introduction to Stochastic Processes. Houghton Mifflin Company, Boston, 1972.