Universitat Autònoma de Barcelona

2019/2020

Linear Models

Code: 100117
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

Name: Mercè Farré Cervelló
Email: Merce.Farre@uab.cat

Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Alejandro Lerer Gornatti

Prerequisites

The student is supposed to know linear algebra, probability theory and statistics, and to have some experience with the R language, but there are not regulated prerequisites.

Objectives and Contextualisation

The objective of the course is to describe, analyze and validate mathematical models that attempt to assess the relationships between different variables under uncertainty conditions. Linear models are probabilistic models that use confidence intervals and statistical hypotheses testing to interpret the results and make decisions. The goal of a regression model is to explain the mean behaviour of a response variable in terms of other variables related to it. Given a model, predictions and residuals can be obtained and analyzed, analysis that will be translated into decisions at an experimental level. The students must be conscious of the constraints in each mathematical model and select which model behaves better. Thus, they must know how to adjust, validate and compare various linear models first, and the subsequent extensions, as generalized-linear or nonlinear models, among others. To this end, theoretical and problems sessions are devoted to explore the theoretical properties of the mathematical models, dealing with data management and modelling using free statistical software in the practical sessions.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Effectively use bibliographies and electronic resources to obtain information.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Effectively use bibliographies and electronic resources to obtain information.
3. Generate innovative and competitive proposals for research and professional activities.
4. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
5. Understand abstract language and understand in-depth demonstrations of some advanced theorems of probability and statistics.

Content

I. Preliminaries.

• The simple linear model: square minimums, maximum likelihood and other estimation methods (inverse, orthogonal, etc.).

• Multidimensional laws: Random vectors. Expectation vector and covariance matrix. Multidimensional normal distribution. Laws related to the Gaussian.

II. The multiple linear model.

• The linear model. Estimable functions. Normal equations. Properties of the coefficients' estimators. BLUE. Estimation of the variance. Goodness of fit measures. The centred model. The model with linear constraints.

• Sum of squares decompositions and distributions. Hypothesis tests and confidence regions. The Cochran theorem. Estimation of the mean response and prediction of new observations.

• Diagnosis of the model: centred, Gaussian, equal variance and uncorrelated errors. Transformations.

• Outliers and influential observations. The multicolinearity problem. The bias problem. Model selection criteria.

III. The analysis of variance and design of experiments.

• One-way analysis of Variance. Multiple comparisons. Diagnostics.

• Analysis of the variance with several factors. Interactions.

• Latin and Greco-Latin squares. Analysis of covariance.

• The 2^2 and 2^k designs. Fractions of factorial designs.

• Response surfaces models.

IV. Certain extensions of the linear model.

• Random effects models. Repeated measures models.

• Generalized linear models: logit, probit, Poisson.

• Nonlinear regression.

Methodology

In the theoretical sessions, the models used will be presented and the mathematical demonstrations and derived properties will be shown. Much emphasis will be placed on applicability and interpretation. The discussion in the classroom will also be encouraged.

In the problem sessions, students will be asked to solve the problems and questions proposed.

In the computer lab sessions, students will work on data modeling with the help of the free R language and environment.

The collaboration and participation of all students will be sought, without discrimination based on sex or any other cause.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Computer work	24	0.96	1, 3, 4, 2
Problems sessions	6	0.24	5, 4, 2
Theoretical classes	30	1.2	5, 2
Type: Autonomous			
Personal work	80	3.2	3, 4, 2

Assessment

The evaluation scheme is as follows:

NC = 0.3 * P1 + 0.4 * P2 + 0.15 * Tb + 0.15 * Lli

P1: First partial exams (30%) = theory and problems (15%) + computer test (15%).

P2: Second partial exams (40%) = theory and problems (20%) + computer test (20%).

Tb: Personal project (15%).

Lli: Delivery of solved problems and practical exercices (15%).

Besides that, the students will have the option of taking an additional recovery exam (RE) with the same format, to recover only the P1+P2 amount.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Personal work	0,15	1	0.04	1, 3, 4, 2
Tasks delivery	0,15	1	0.04	3, 4, 2
Two partial computer tests	0,35	4	0.16	1, 4, 2
Two partial exams (theory and problems)	0,35	4	0.16	5, 4, 2

Bibliography

- Peña, D.; Regresión y Diseño de Experimentos. Alianza Editorial, 2002.
- Rao, C. B., Touttenburg, H., Shalabh, Heumann, C.; Linear Models and Generalizations. 3rd edition, Springer, 2008.
- Rawlings J. O, Pantula S. G , Dickey D. A.; Applied Regression Analysis. A Research Tool. Second Edition, Springer, 1999.
- Rencher, A.C., Schaalje, G.B.; Linear Models in Statistics. Wiley-Interscience, 2008.
- Seber, G., Lee, A.; Linear Regression Analysis. Wiley Series in Probability and Statistics, 2003.
- Hay-Jahans C.; An R Companion to Linear Statistical Models. Chapman and Hall, 2012.
- Faraway, J.; Linear Models with R. Chapman&Hall/CRC, 2005.
- Faraway, J.; Extending the linear model with R. Chapman&Hall/CRC, 2006.
- Vikneswaran; An R Companion to Experimental Design. https://cran.r-project.org/doc/contrib/Vikneswaran-ED_companion.pdf

Complementary references

- McCullagh, P., Nelder, J. A.; Generalized Linear Models. Chapman&Hall, 1989.
- Clarke, B. R.; Linear Models. The theory and applications of Analysis of Variance. Wiley Series in Probability and Statistics, 2008.
- Sen, A., Srivastava, M.; Regression Analysis. Theory, Methods and Applications. Springer, 1990.
- Carmona, F.; Modelos Lineales. Universitat de Barcelona, 2005.
- Christensen, R.; Advanced Linear Modelling. Springer, 2001.
- Christensen, R.: Log-Linear Models. Springer, 1990.
- Draper, N., Smith, H.; Applied regression Analysis. Wiley, 1998.
- Chatterjee, S. & Price, B.; Regression Analysis by Example. Wiley-Interscience, third edition, 2000.
- Scheffé, H.; The Analysis of Variance, 1999.
- Montgomery, D.C., Peck, E., Vining, G.; Introduction to Linear Regression. Wiley Series in Probability and Statistics, 2001.