Universitat Autònoma de Barcelona

2019/2020

Partial differential equations

Code: 100119
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

Name: Xavier Mora Giné
Email: Xavier.Mora@uab.cat

Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Angel Calsina Ballesta

Prerequisites

Desirable: previous courses of ordinary differential equations and vector calculus. ALso advantageous: general physics.

Objectives and Contextualisation

The course of Partial Differential Equations will be devoted to study and expand the knowledge of one of the most important mathematical tools in the applications of mathematics to science and technology. Based on the skills acquired in the course of Differential Equations and Modeling II, we will make a general introduction of some of the most important partial differential equations in the historical development of mathematics and physics, at the same time that we will remember some tools of Vector Calculus.

After this, the first major aim of the course will be the nonlinear equations of first order, such as conservation laws. With this goal we will study first the most basic aspects of the method of characteristics for quasi-linear equations. Some applications of these models, such as the traffic equation, will be used to visualize the difficulties of modeling and the meaningfulness of solutions in a generalized sense, such as shocks and rarefaction waves.

The other main aim of the course will be the basic aspects of the "typical" second-order linear equations of mathematical physics: potential, heat and waves.

Competences

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.

- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Understand and use mathematical language.

Learning Outcomes

1. Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
2. Know how to demonstrate the results of partial derivative equations and dynamical systems.
3. Know how to solve certain theoretical problems and be understand the existence of certain open problems in the theory of partial derivative equations and dynamical systems theory.
4. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
5. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.

Content

1. Introduction to partial differential equations

1.1. Equations in partial derivatives to Science, Technology and Finance.
1.2. Basic concepts: order, linearity.
1.3. Vector Analysis Elements: Differential Operators. The theorems of Green, of the divergence of Gauss and Stokes.
1.4. Equations of Physics-Mathematics: the equation of heat, the equation of waves, the equation of potential. Initial conditions and outline conditions. Stationary problems.

2. Partial differential equations of first order

2.1. Llinear and quasilinear partial differential equations of first order with two variables. The method of characteristics. The initial value problem.
2.2. Introduction to conservation laws. The traffic equation. Some initial value problems. Rarefaction waves and and shocks. Entropy condition.
2.3. Nonlinear equations of first order.

3. Semilinear partial differential equations of second order

3.1. Canonical forms of second-order semilinear equations with two variables. Classification

4. The wave equation

4.1. One-dimensional waves. Alembert's formula. Areas of influence and dependence. Reflections
4.2. The equation of the waves in dimensions 2 and 3.

5. The heat equation

5.1. The heat equation in space. The Poisson formula. Regularity
5.2. The maximum principle. Uniqueness of solution.

6. The potential equation

6.1. Harmonic functions. Properties.
6.2. The problems of Dirichlet and Neumann. Uniqueness of solution
6.3. Functions of Green
6.4. The principle of Dirichlet and variational methods

Methodology

This course consists of 2 hours a week of lectures, 1 hour a week of problem solving and three seminars of two hours each.

The lectures will explain the different types of partial differential equations, their derivation, the characteristic properties of each type of equation, the methods of resolution and the various concepts of solution.

The problem sessions will be devoted to solving problems of a practical nature that students will develop on the board. For this, work will be done on lists of problems that will be previously provided to the student.

The seminars will focus on specific issues and the guided solution of more complex problems.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	30	1.2	
Type: Supervised			
Problem sessions and working seminars	21	0.84	
Type: Autonomous			
Problem solving	34	1.36	
Studying theoretical concepts	50	2	

Assessment

In principle, the course will be evaluated by means of two partial exams, each of them counting 40%, and the assessment of the seminars, which will count 20%. However, this is subject to the condition that both marks of the partial exams be higher than or equal to 3.5. If this condition is not met, or if the obatined global mark is less than 5, then the student can opt for a supplementary exam that will replace the two partial ones and will count 80%. The evaluation of the seminars is not recoverable.

The possible honor qualifications will be assigned based on basis of the global notes that result from the two partial exams and the seminars, that is to say, without waiting for the supplementary exam. If this does not exhaust the number of honor available honor qualifications, then the remaining ones may be assigned after the supplementary exam.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	40%	4.5	0.18	3, 1, 5, 4, 2
Second partial exam	40%	4.5	0.18	3, 1, 5, 4, 2
Seminars	20%	6	0.24	3, 1, 5, 4, 2

Bibliography

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J. Ockendon, S. Howison, A. Lacey, A. Movchan, *Applied partial differential equations.* Oxford University Press, 2003

Y. Pinchover, J. Rubinstein *An Introduction to Partial Differential Equations, Cambridge, 2005.*

M. Renardy, R. C. Rogers, *An Introduction to partial differential equations*, Springer, 2004.
S. Salsa, *Partial Differential Equations in action: from modelling to theory,* Springer, 2008.
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E.H. Zauderer, *Partial differential equations of applied mathematics*, Wiley-Interscience, 2006.