

**Advanced Mathematical Methods**

Code: 100167  
ECTS Credits: 5

Degree	Type	Year	Semester
2500097 Physics	OT	3	1

**Contact**

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**Use of Languages**

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: No  
Some groups entirely in Spanish: No

**Teachers**

Santiago Perís Rodríguez

**Prerequisites**

It is advisable to have studied the following subjects:  
Calculus in one variable  
Vector Calculus  
Differential equations

**Objectives and Contextualisation**

This subject introduces some basic mathematical concepts  
needed in physics in general, and in physics / Quantum mechanics  
and field theories, in particular. It is intended that the student  
achieve the understanding of the concepts of Hilbert space, operators,  
and, especially, groups. It wants to give an integrative vision  
of concepts that appear in different fields in physics. At the same time,  
the student will have to acquire the capacity to apply them with agility  
for different types of problems.

**Competences**

- Apply fundamental principles to the qualitative and quantitative study of various specific areas in physics

- Be familiar with the bases of certain advanced topics, including current developments on the parameters of physics that one could subsequently develop more fully
- Develop critical thinking and reasoning and know how to communicate effectively both in the first language(s) and others
- Develop independent learning strategies
- Develop the capacity for analysis and synthesis that allows the acquisition of knowledge and skills in different fields of physics, and apply to these fields the skills inherent within the degree of physics, contributing innovative and competitive proposals.
- Generate innovative and competitive proposals for research and professional activities.
- Respect the diversity and plurality of ideas, people and situations
- Use critical reasoning, show analytical skills, correctly use technical language and develop logical arguments
- Use mathematics to describe the physical world, selecting appropriate tools, building appropriate models, interpreting and comparing results critically with experimentation and observation

## Learning Outcomes

1. Classify representations of the most simple groups.
2. Determine the effect on the observables of symmetry transformation.
3. Determine the observables that characterise representation.
4. Determine the representation that characterizes a particular physical system.
5. Determine the symmetry group (exact or approximate) associated with a physical system.
6. Develop critical thinking and reasoning and communicate ideas effectively, both in the mother tongue and in other languages.
7. Develop independent learning strategies.
8. Generate innovative and competitive proposals for research and professional activities.
9. Identify symmetry groups associated with the laws of physics.
10. Identify symmetry groups associated with theories of fundamental interactions.
11. Identify symmetry groups in addition to their particle and crystallography representations, associated with atomic physics.
12. Obtain representation of simple symmetry groups.
13. Relate continuous groups with the Lie algebra to which they are associated.
14. Relate the symmetries of nature with the appropriate symmetry group (exact or approximate).
15. Respect diversity in ideas, people and situations.
16. Use critical reasoning, show analytical skills, correctly use technical language and develop logical arguments
17. Use the tensor calculus.

## Content

### PROGRAM

1. Hilbert spaces
  - 1.1 Pre-Hilbert spaces.
  - 2.2 Hilbert spaces.
2. Operators.
  - 2.1 Linear operators.
  - 2.2 Eigenvalues and eigenvectors.
3. Introduction to group theory
  - 3.1 Definition and motivation (symmetries)

- 3.2 Exemples:  $SO(3)$ ,  $SU(2)$ ,  $SU(N)$  (relation with unitary operators).
- 3.3 Lie algebras (generators of the continuous group)
- 3.4  $\mathfrak{su}(N)$  (relation with selfadjoint operators) and relation with  $\mathfrak{su}(2)$  with  $\mathfrak{so}(3)$
- 3.5 Representations (vocabulary)
- 3.6 Tensorial methods
- 3.7 Applications:  $SU(N)$ , discrete groups and Lorentz group

## Methodology

This course develops mathematical language and calculation tools that are basic for advanced physics subjects. The personal work of the student is fundamental. Classroom sessions will be divided into:

Lectures: The teacher will present the basic concepts and reasoning of the Subject, with the support of examples.

Problem classes: Among a collection of problems, the teacher will solve in detail a selection. Students will have to work on their own the

## Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Blackboard lectures: the professor will expose basic concepts and arguments for each subject, with the support of detailed examples.	27	1.08	
Practical lectures: among a problems collection, the professor will solve a set of them. The rest will be solved by the students.	14	0.56	
Type: Autonomous			
Individual and groupal work solving problems	28	1.12	
Selective homework	11	0.44	
Study of theoretical foundations	37	1.48	

## Assessment

Partial examination of Hilbert Spaces and operators: 45% of the note.

Group partial exam: 50% of the mark.

Selective delivery of problems: 5% of the note.

By imperative of the general regulations of the university, to be able to pass the exam, the student must obtain a minimum mark of 50%.

Resit exam of the two partials: 95% of the note. There is no minimum mark.

## Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Group theory exam	50%	2.5	0.1	1, 7, 6, 5, 3, 2, 4, 8, 9, 10, 11, 12, 16, 13, 14, 15, 17
Hilbert spaces and operators exam	45%	2.25	0.09	7, 6, 8, 16, 15, 17
Homework	5%	0.25	0.01	1, 7, 6, 5, 3, 2, 4, 8, 9, 10, 11, 12, 16, 13, 14, 15, 17
Resit exam	95%	3	0.12	1, 7, 6, 5, 3, 2, 4, 8, 9, 10, 11, 12, 16, 13, 14, 15, 17

## Bibliography

Basic bibliography.

P. Szekeres, *A course in Modern Mathematical Physics*.

Elvira Romera et al., Métodos matemáticos: Problemas de espacios de Hilbert, operadores lineales y espectros

G. Arfken, *Mathematical Methods for Physics*.

Advanced and complementary bibliography.

J.J. Sakurai, *Modern Quantum Mechanics*.

J.F. Cornwell, *Group theory in Physics*.

H. Georgi, *Lie Algebras in particle physics*.

L. Abellanas i A. Galindo, *Espais de Hilbert*.

S.K. Barbarian, *Introducció a l'espai de Hilbert*.

L. Schwartz, *Métodos Matemáticos para las ciencias físicas*.