2019/2020

Partial Different Equations: Modelling, Analysis and Numerical Approximation

Code: 44211
ECTS Credits: 6

Degree	Type	Year	Semester
4313136 Modelling for Science and Engineering	OT	0	2

Contact

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Use of Languages

Principal working language: catalan (cat)

Teachers

Joan Carles Artés Ferragud

Prerequisites

Students should have basic knowledge of calculation, algebra, ordinary and partial differential equations, and bas

Objectives and Contextualisation

Partial differential equations allow deterministic mathematical formulations of phenomena in physics and enginee

present the main results in the context of partial differential equations that allow learning about these models and

Competences

- Analyse, synthesise, organise and plan projects in the field of study.
- Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
- Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
- Apply techniques for solving mathematical models and their real implementation problems.
- Formulate, analyse and validate mathematical models of practical problems in different fields.
- Isolate the main difficulty in a complex problem from other, less important issues.
- Present study results in English.
- Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
- Use appropriate numerical methods to solve specific problems.

Learning Outcomes

1. Analyse, synthesise, organise and plan projects in the field of study.
2. Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
3. Apply partial derivative equation techniques to predict the behaviour of certain phenomena.
4. Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
5. Extract information from partial derivative models in order to interpret reality.
6. Identify real phenomena as models of partial derivative equations.
7. Isolate the main difficulty in a complex problem from other, less important issues.
8. Present study results in English.
9. Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
10. Solve real problems by identifying them appropriately from the perspective of partial derivative equations.
11. Use appropriate numerical methods to study phenomena modelled with partial derivative equations.

Content

Introduction: General classification of partial differential equations, examples of models. Transport equation, method of characteristics.

1. Parabolic equations

Fourier method. Heat equation. Fundamental solution, Gaussian kernel, convolution and solution formula for the pure initial value problem. Maximum principle and uniqueness of the solution. Numerical Methods: Finite difference methods for scalar parabolic equations: Euler Explicit, Euler Implicit and Crank-Nicolson methods: Von Neumann stability test. Parabolic stability CFL condition. Examples

2. Eliptic equations

Theory: Steady-state problems. Polar/Spherical coordinates: radial solutions. Dirichlet and Neumann boundary value problems. Poisson kernel. Applications. Euler-Lagrange equations associated to variational problems. Numerics and examples.

3. Hyperbolic equations

Scalar Conservation Laws. Weak solutions. Burgers equation. Shock waves and expansions fans. Hamilton-Jacobi equations and viscosity solutions. Introduction to the Level Set Method. Eikonal equation.

Numerical Methods: Finite difference methods in conservation form. Shock-capturing schemes. Monotone schemes: Lax-Friedrichs and upwind schemes. Convergence and stability conditions. Entropy-satisfying schemes. Examples. Level set method applications.

Methodology

The objective of the classes of theory, problems and practices is to give to the students the most basic knowledg

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Classes of theory and problems	30	1.2	5, 6, 10
Type: Supervised			
Internship classes	8	0.32	11
Type: Autonomous			
Studies and practical work by the student.	96	3.84	5, 6, 10

Assessment

The assessment will consist of two partial exams and the delivery of the resolution of a problem through the com

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	40%	4	0.16	2, 1, 4, 3, 8, 7, 5, 6, 9, 10, 11
Second partial exam	40%	4	0.16	10
Solution of a problem with a computer	20%	8	0.32	2, 1, 4, 3, 8, 7, 5, 6, 9, 10

Bibliography

L.C. Evans, Partial differential equations, Graduate Studies in Mathematics 19 (2nd ed.), Providence, R.I., American Mathematical Society, (2010).

B. Gustafson, H-O. Kreiss and J. Oliger, Time dependent problems and Difference Methods, Wiley-Intersciences, (1996).

F. John, Partial Differential equations, vol. 1, Applied Math Sciences, Springer, (1978).

P.D. Lax, Hyperbolic systems of Conservation Laws and The Mathematical Theory of Shock Waves SIAM, 1973.

R.J. LeVeque, Finite Volume Methods for Hyperbolic problems, Cambridge University Press, 2002.

Y.Pinchover, J. Rubinstein, An Introduction to Partial Differential Equations, Cambridge 2005.

S. Salsa, Partial differential equations in action : from modelling to theory Springer, 2008.

G. Strang, Introduction to Applied Mathematics, Wellesley-Cambridge Press, (1986).

E.F. Toro Riemann Solvers and Numerical Methods for Fluid Dynamics: A practical Introduction, Springer-Verlag, 2009.

G.B. Whitham Linear and nonlinear Waves, Wiley-Intersciences, (1999).