

Fundamental Mathematics

Code: 100089
ECTS Credits: 9

| Degree | Type | Year | Semester |
|---------------------|------|------|----------|
| 2500149 Mathematics | OB | 1 | 1 |

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

Contact

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Use of Languages

Principal working language: spanish (spa)
Some groups entirely in English: No
Some groups entirely in Catalan: No
Some groups entirely in Spanish: No

Teachers

Pere Ara Bertrán
Laia Saumell Ariño
Francesc Perera Domènech
Wolfgang Pitsch

Prerequisites

Apart from a good understanding of basic notions in arithmetic and some skill in handling algebraic expressions, no prerequisites are needed for this course. Nonetheless it is basic to have the will to understand the mathematical arguments, the logic and to sharpen one's critical thinking.

Objectives and Contextualisation

In the first part of the course we will introduce the basic language of mathematics. A great deal of time will be dedicated to getting to handle this new language correctly, as it is essential to understand, produce and share mathematics.

Particular stress will be put on the logic arguments (implication, equivalence, contraposition). The student will get acquainted to these through the diverse themes of the course: basic set theory, arithmetic, polynomials, etc.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.

- Calculate and reproduce certain mathematical routines and processes with agility.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Acquire basic training to be able read the headings of results and their demonstrations, identify situations in which counter-examples are necessary.
2. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
3. Adapt theoretical reasoning to new demonstrations and situations.
4. Apply critical spirit and thoroughness to validate or reject both ones own arguments and those of others.
5. Deal with the basic concepts of set theory as shown in the table of contents.
6. Resolve congruencies and calculate roots of polynomials
7. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
9. Understand equivalence and order ratios.
10. Understand quotient sets and work with them.
11. Understand some demonstration methods.
12. Understand the basic concept of application and know how to apply it.
13. Use symbolic computation to resolve congruencies and decompose polynomials.
14. Use the methods of some demonstrations to make specific calculations: resolution of Diophantine equations and congruence equations, factorisation of polynomials if any root is known

Content

1. Set theory

Complex numbers

Basic language of sets.

Peano Axioms. Induction.

Maps between sets. Equivalence and order relations. Quotient set.

Permutations. Decomposition in disjoint cycles, order and sign.

2- Combinatorics

Finite vs infinite sets.

(Un)Ordered selections, with and without repetition.

Binomial formula.

Inclusion-exclusion principle.

3. Integers and congruences

Euclidean division. Greatest common divisor and least common multiple. Bézout Identity.

Diophantine Equations.

Prime and coprime numbers. Factorization.

Congruences. Euler and Fermat theorems. Chinese reminder theorem.

4. Polynomials

Euclidian division in polynomials. Greatest common divisor and least common multiple. Bézout Identity. Irreducible polynomials and coprime polynomials. Factorization into irreducibles. Roots.

Methodology

There are three type of activities the student is supposed to attend: the lectures (3 hours /week) mainly concerned with the description of the theoretical concepts, problem solving sessions (1 hour/week) and seminars (2 hours on alternate weeks), similar to the problem solving sessions but where students work in groups supervised by a teaching assistant. The course has a web page in the UAB online campus gathering all information and communications between students and professors, and where all material, including problem sheets, solutions, etc are published regularly. Students are supposed to submit three exercises sets in three of the working seminars. These exercises will be graded and returned to the students.

The methodology and the activities are adapted to the training objectives of the course: introduce the mathematical language, learn to use it correctly, see demonstrations and demonstration methods. To achieve the objectives it is important that the first-year student sees and understands the development of the theory but also, and may be above all, that she/he tries to do the exercises, writing them correctly, imitating what she/he has seen in theory classes.

It must be borne in mind that the correct assimilation of the syllabus of this subject requires dedication, continuous and sustained work on the part of the student. In an indicative way, you would have to work on a personal basis as many hours a week as class hours have the subject. In case of doubts it is important to consult with the professors, both in theory and in problem.

Activities

| Title | Hours | ECTS | Learning Outcomes |
|--|-------|------|---|
| Type: Directed | | | |
| Lectures | 43 | 1.72 | 4, 11, 12, 10, 5, 13 |
| Problem session | 27 | 1.08 | 8, 7 |
| Type: Supervised | | | |
| Working seminars | 20 | 0.8 | |
| Type: Autonomous | | | |
| Studying theoretical concepts and solving problems | 122 | 4.88 | 3, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |

Assessment

You will be evaluated according to the following guidelines.

1) Homework. you will be asked to hand over to the Teacher periodic homework. This is mandatory. The homework counts for 15% of the total grade.

2) Seminars. You will be asked to complete some ambitious problems with the guidance of the seminar professors. Some students may be asked to explain orally their solutions. This will count for 15% of the final

grade.

3) Submission of exercise sets through the virtual platform ACME. The deadlines will be announced in intranet of the course. Counts for 10% of the final grade.

4) Mid term exam. 30% of the final grade.

5) Final Exam. 30% of the final grade.

To be allowed pass the mean grade of the Mid-term Exam and the Final Exam have to be at least 3.5. If not, that mean will be your final grade.

6) For those students that score less than 5 after the final exam, there will be a reevaluation. the grade of your reevaluation exam will replace that of the partial and final exams.

7) La qualification of "Not assessable" will be given to those students not attending the final exam.

8) Honors qualifications may be given after the final exam.

Assessment Activities

| Title | Weighting | Hours | ECTS | Learning Outcomes |
|---|-----------|-------|------|--|
| Electronic submission of exercise sets (ACME) | 10% | 1 | 0.04 | 3, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |
| Final test | 30% | 3 | 0.12 | 3, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |
| Homework assignments | 15% | 0 | 0 | 3, 1, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |
| Mid-term test | 30% | 3 | 0.12 | 3, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |
| Reevaluation exam | 60% | 3 | 0.12 | 3, 4, 11, 12, 10, 9, 5, 8, 7, 6, 13, 14 |
| Seminar | 15% | 3 | 0.12 | 4, 11, 2, 12, 10, 9, 5, 8, 7, 13, 14 |

Bibliography

A comprehensive set of course notes, translated and completed from "An Introduction to Proof via Inquiry-Based Learning" by Dana C.~Ernst will be provided to the students.

Bibliography

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A. Cupillari, *The nuts and bolts of proofs*. Elsevier Academic Press, 2005.

E. Bujalance, J.A. Bujalance, A.F. Costa, E. Martínez. *Problemas de Matemática Discreta*. Sanz y Torres, Madrid.

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P.J. Eccles, *An introduction to mathematical reasoning, numbers, sets and functions*. Cambridge University Press, Cambridge, 2007.

A. Reventós, *Temes diversos de fonaments de les matemàtiques*. Apunts.

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