

Algebraic structures

Code: 100096
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	2

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Francesc Xavier Xarles Ribas
Laurent Cantier

Prerequisites

The previous academic requirements will be found in the subjects Fundamentals of Mathematics and Linear Algebra, first year.

The skill acquired in algebraic manipulations, and the familiarity with operations in arithmetic contexts or groups of permutations, will continue to be developed, moving to a higher level of abstraction, which is very common in Mathematics. References to vector spaces as a model of algebraic structure and to your knowledge of matrix manipulation will also be frequent. Matrices will be a particularly important source of examples.

Objectives and Contextualisation

The objectives of this subject are of two types: to achieve training in basic algebra and gaining knowledge and skills to manipulate abstract objects.

Among the training objectives we highlight the following:
correctly understand and use language and mathematical reasoning in general and algebraic reasoning in particular. Be able to make small demonstrations, develop meaning
critical of mathematical statements,
develop combative attitudes and creativity in the face of problems and, finally, learn to apply abstract concepts and results in concrete examples. Present reasoning or a problem in public and develop agility to answer mathematical questions in a conversation.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Calculate the maximum common divisor and factorisation of whole numbers and polynomials.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Operate in some simple groups (such as cyclic, dihedral, symmetric and abelian).
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
7. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

The subject is organized in four parts:

I. Group Theory.

- Groups, subgroups and morphisms. Basic examples.
- Lateral classes. Lagrange Theorem.
- Normal subgroups, quotient group.
- Isomorphism theorems.
- Classification of cyclic groups. More on abelian groups.
- Action of a group on a set.
- Sylow's theorems.

II. Commutative rings

- Rings, ideals and morphisms. Basic examples
- Quotients and isomorphism theorems.

- Maximal and prime ideals. Zorn's Lemma.
- Field of fractions of a domain.
- The ring of polynomials

III. Factorization.

- Domains of main ideals.
- Unique factorization domains.
- Gaussian lemma. Factorization in rings of polynomials.

IV. Finite fields.

- Fields, subfields and characteristic of a field.
- Primitive element theorem for finite fields.
- Existence and uniqueness of finite fields.
- Frobenius morphism.

Methodology

This subject has three ours per week of theory lasses, one hour per wek of problem classes, and, during the semester, eight seminar sessions, two ours each.

Students will have the lists of problems previously to be able to work before the problem classes. In classe, you can not solve all the problems but we recommend that students work on their own and asj the teachers their questions. In the seminar sessions the students will work under the supervision of the teacher. In some of these seminars, some exercises will be given that will count for the final mark of the subject.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Directed	16	0.64	
Theory classes	43	1.72	
Type: Supervised			
Sminars	14	0.56	
Type: Autonomous			
Seminar preparation	145	5.8	

Assessment

- The student will obtain 10% of the mark of the subject with the delivery of exercises previously done. We denote by LP the mark on 10 calculated as the average of deliveries.

-A written examination will be carried out to evaluate the theoretical and practical knowledge of the subject in mid-semester. The mark on 10 (P1) of this examination will count 30% of the mark of the course.

-In some of the seminars, classroom exercises will be given. They will be short exercises to evaluate practical aspects of the newly completed seminar. The mark (S) on 10, calculated as the average of the seminar marks, will count 10% of the marks of the subject.

-50% of the mark of the subject will correspond to the mark P2 obtained in the final examination. This examination will evaluate the student's practical and theoretical knowledge.

The mark of the subject is obtained by the expression $N = 0,10.LP + 0,10.S + 0,30.P1 + 0,50.P2$. The student will pass if N is greater or equal than 5.

The qualification of Excellent with honours will be awarded based on N

There will be a resit examination corresponding to the final examination. Only students with the mark N less than 5 and who have been presented to the examination that give rise to the P1 and P2 marks may attend the resit examination. In this case, the final mark of the subject will be calculated as $\text{Max}(N; 0,10.LP + 0,10.S + 0,30.P1 + 0,50.R)$ where R denotes the mark of examination resit

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Autonomous	145 hours , 5,8 ECTs	4	0.16	2, 3, 1, 4, 8, 7, 6, 5
Supervised	14 hours, 0,56 ECTS	3	0.12	2, 3, 1, 4, 8, 7, 6, 5

Bibliography

- [1] R. Antoine, R. Camps, J. Moncasi. Introducció a l'àlgebra abstracta. Manuals de la UAB, Servei de Publicacions de la UAB, nº 46, Bellaterra, 2007.
- [2] F. Cedó, V. Gisin, Àlgebra bàsica, Manuals de la UAB, Servei de Publicacions de la UAB, nº 21, 2007.
- [3] P. M. Cohn, Algebra, vols. 1 i 2, John Wiley and Sons, 1989.
- [4] J. Dorronsoro, E. Henández, Números, Grupos y anillos, Addison-Wesley, 1996.
- [5] F. Delgado, C. Fuertes, S. Xambó, Introducción al Álgebra: anillos, factorización y teoría de cuerpos, Universidad de Valladolid, 1998.
- [6] J. B. Fraleigh, A First course in Abstract algebra, Addison-Wesley, 1982.
- [7] T. W. Hungerford, Álgebra, Springer-Verlag, 1974.