

Real and functional analysis

Code: 100110
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Prerequisites

All the previous courses of Calculus and Mathematical Analysis.

Good knowledge of Linear Algebra and Basic Topology is also important.

Objectives and Contextualisation

Explain the concepts and fundamental results of the Lebesgue integral in Euclidean space.

Present the methods of functional analysis, in the context of Banach spaces and Hilbert spaces.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
3. Confidently deal with the most important Hilbert spaces and know how to apply the basic theory of Functional Analysis to them.
4. Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
5. Devise demonstrations of mathematical results in the field of mathematical analysis.
6. Effectively use bibliographies and electronic resources to obtain information.
7. Formulate conjectures and devise strategies to confirm or reject said conjectures
8. Generate innovative and competitive proposals for research and professional activities.
9. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
10. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
11. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
12. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
13. Understand the concept of R^n measurement and its construction process.
14. Understand the language and in-depth demonstrations of some advanced mathematical analysis theorems.
15. Understand the nature of the Lebesgue integral and its advantages over the Riemann integral.

Content

The course consists of 3 blocks:

Theory of Measure, Banach Spaces and Hilbert Spaces.

1. Limitations of the Riemann integral.
2. Lebesgue measure. Abstract measure theory.
3. Lebesgue integral. Abstract integral theory. Limit vs. integral.
4. Fundamental Theorem of Calculus. Variable change theorem. Fubini-Tonelli theorem.
5. Integrals dependent on a parameter. Differentiating under the integral sign.
6. Normed spaces. Banach spaces. Characteristics.
7. Spaces of sequences. Spaces of functions. Spaces of measures.
8. Bounded linear operators. Norm of an operator. Topology of bounded linear operators.
9. Applications: Volterra's integral equation.
10. Open Mapping Theorem and Closed Graph Theorem. Uniform boundedness principle.

11. Dual topological of a normed space. Hahn-Banach theorem.
12. Hilbert spaces. Theorem of the Projection. Orthogonality
13. Hilbertian basis. Bessel inequality. Parseval's identity.
14. Fourier series. Riemann-Lebesgue lemma.
15. Compact operators. Sturm-Liouville problem.

Methodology

This subject has 2 hours of theory and 1 of problems per week.

It also consists of a total of 6 hours of seminars throughout the course.

Although it is not compulsory, it is highly recommended to attend classes to ask questions and venture answers, even if they are incorrect.

Theory: we will develop the main results and put them in the context of future applications.

Problems: students will receive some lists of exercises that we will solve in problem classes.

Seminars: will serve to complement the contents of theory and problems.

Students will also have a few hours of consultation in the teacher's office, to consult questions, discuss methods, etc.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercices lessons	14	0.56	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Theoretical lessons	30	1.2	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Type: Supervised			
Seminars	6	0.24	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Type: Autonomous			
Personal study	92	3.68	

Assessment

During the course we will do an evaluation activity (two hours) for each block. It will consist in presenting the demonstration of some result, of a list established before the evaluation, and in the resolution of exercises.

Block 1. Measure Theory (30%)

Block 2. Banach Spaces (30%)

Block 3. Hilbert Spaces (30%)

The delivery of solved exercises, as the teacher did, indicating, complements (10%) the course evaluation.

On the day designated by the Coordination of the Degree as a Final Exam (or recovery), students who have not passed the course will take a make-up exam with all the course material. The maximum score that can be obtained in this recovery exam is 7.

ALL THE CONTENTS OF THE COURSE ARE EVALUABLE (THEORY, PROBLEMS, SEMINARS).

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Block 1. Measure Theory	30%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 10, 6
Block 2. Banach Spaces.	30%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6
Block 3. Hilbert Spaces	30%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6
Delivery of exercises	10%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6

Bibliography

J. Bruna, Anàlisi Real, UAB Servei de Publicacions, 1996.

J.M. Burgués, Integració i càlcul vectorial, UAB Servei de Publicacions, segona edició, 2002.

S. Lang, Real and functional analysis, Graduate texts in mathematics, Springer, 1993.

W. Rudin Real and functional analysis, Alambra, 1979.