Universitat Autònoma de Barcelona

2021/2022

Algebraic structures

Code: 100096
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	2

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Francesc Xavier Xarles Ribas

Prerequisites

The previous academic requirements will be found in the subjects Fundamentals of Mathematics and Linear Algebra, first year.

The skill acquired in algebraic manipulations, and the familiarity with operations in arithmetic contexts or groups of permutations, will continue to be developed, moving to a higher level of abstraction, which is very common in Mathematics. References to vector spaces as a model of algebraic structure and to your knowledge of matrix manipulation will also be frequent. Matrices will be a particularly important source of examples.

Objectives and Contextualisation

The objectives of this subject are of two types: to achieve training in basic algebra and gaining knowledge and skills to manipulate abstract objects.

Among the training objectives we highlight the following:
correctly understand and use language and mathematical reasoning in general and algebraic reasoning in particular. Be able to make small demonstrations, develop meaning
critical of mathematical statements,
develop combative attitudes and creativity in the face of problems and, finally, learn to apply abstract concepts and results in concrete examples. Present reasoning or a problem in public and develop agility to answer mathematical questions in a conversation.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Calculate the maximum common divisor and factorisation of whole numbers and polynomials.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Operate in some simple groups (such as cyclic, dihedral, symmetric and abelian).
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
7. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

The subject is organized in four parts:

I. Group Theory.

- Groups, subgroups and morphisms. Basic examples.
- Lateral classes. Lagrange Theorem.
- Normal subgroups, quotient group.
- Isomorphism theorems.
- Classification of cyclic groups. More on abelian groups.
- Action of a group on a set.
- Sylow's theorems.

II. Commutative rings

- Rings, ideals and morphisms. Basic examples
- Quotients and isomorphism theorems.
- Maximal and prime ideals. Zorn's Lemma.
- Field of fractions of a domain.

- The ring of polynomials

III. Factorization.

- Domains of main ideals.
- Unique factorization domains.
- Gaussian lemma. Factorization in rings of polynomials.

IV. Finite fields.

- Fields, subfieds and characteristic of a field.
- Primitive element theorem for finite fields.
- Existence and uniqueness of finite fields.
- Frobenius morphism.

Methodology

This subject has three ours per week of theory lasses, one hour per wek of problem classes, and, during the semester, eight seminar sessions, two ours each.

Students will have the lists of problems previously to be able to work before the problem classes. In classe, you can not solve all the problems but we recommend that students work on their own and asj the teachers their questions. In the seminar sessions the students will work under the supervison of the teacher. In some of these seminars, some exercises will be given that will count for the final mark of the subject.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Directed	16	0.64	
Theory classes	43	1.72	
Type: Supervised			
Sminars	14	0.56	
Type: Autonomous			
Seminar preparation	145	5.8	

Assessment

20% of the mark of the course corresponds to the delivery of problems to the seminars (S).

There will be a written test, in the middle of the semester, to evaluate the
50% of the mark corresponds to that obtained in the final exam (F). In th

the theoretical and practical knowledge of the subject will be evaluated.
If in F a mark greater than or equal to 3.5 is obtained, then the student o

Honor registrations will be awarded based on the value of the mark N.
There will be a recuperation exam corresponding to the final exam. Only

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Autonomous	145 hours , 5,8 ECTs	4	0.16	2, 3, 1, 4, 8, 7, 6, 5
Supervised	14 hours, 0,56 ECTS	3	0.12	2, 3, 1, 4, 8, 7, 6, 5

Bibliography

[1] R. Antoine, R. Camps, J. Moncasi. Introducció a l'àlgebra abstracta. Manuals de la UAB, Servei de Publicacions de la UAB, no. 46, Bellaterra, 2007.

[2] F. Cedó, V. Gisin, Àlgebra bàsica, Manuals de la UAB, Servei de Publicacions de la UAB, no. 21, Bellaterra, 2007.

[3] David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd. Edition, Wiley, 2003.

[4] J.B. Fraleigh. A First course in abstract algebra. Pearson Education, 7th Edition, 2014. Review: https://www.maa.org/press/maa-reviews/abstract-algebra

[5] T. W. Hungerford, Abstract Algebra, Brooks/Cole, 2013. Review:

https://www.maa.org/press/maa-reviews/abstract-algebra-an-introduction

Software

It is not planned to use any specific software in the subject. However, an algebraic manipulator (Maple, Sage, ...)
There are specific programs for manipulating groups such as GAP - Grou
a System for Computational Discrete Algebra that is useful to know and t