

**Differential equations and modelling I**

Code: 100100  
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	1

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

### Contact

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### Use of Languages

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: Yes  
Some groups entirely in Spanish: No

### Teachers

Joan Carles Artés Ferragud

### Prerequisites

Linear Algebra

Calculus in Several variables.

### Objectives and Contextualisation

The Theory of Differential Equations is distinguished both by the richness of ideas and methods as well as by its applicability. Thus the subject Differential Equations and Modeling I has a theoretical aspect (that will be used in theory and problem lessons) as well as a very applied aspect (which will be introduced in the theory sessions and will be developed in problems and practical sessions). Practical lessons will be carried out in the computer lab. On the one hand we will emphasize the presentation of the theory and the demonstration of the results and on the other hand the students will learn how to model real situations that allow them to predict the studied behaviors.

We think that this subject is good to show to the students that certain theoretical results that they already know about other subjects (topological properties of normed spaces and Jordan canonical forms, for example) can be applied to develop the theory of differential equations.

### Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Recognise the presence of Mathematics in other disciplines.

- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.
- Work in teams.

## Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Apply the main methods for resolving ordinary differential equations and some simple partial derivative equations.
3. Resolve linear systems of ordinary differential equations.
4. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
5. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
6. Translate some real problems into the terms of ordinary differential equations and partial derivative equations.
7. Work in teams

## Content

1. Differential Equations of the first order.
  - 1.1 Introduction to differentia equations. Separable equatiuons. Exact equations.
  - 1.2 Applications to modelling.
2. The linear equation.
  - 2.1 Uniqueness and existence theorems. Algebraic properties of the space of solutions. Liouville's theoreme.
  - 2.2 The autonomous case: Exponential of a matrix.
  - 2.3 The linear equation of order n.
3. Uniqueness and existence theorems
  - 3.1 The Cauchy's problem. Picard and Peano theorems.
  - 3.2 Prolongation of solutions. Wintner's lemma
  - 3.3 Continuous and differentiable dependence on initial data and parameters.
4. Qualitative theory of autonomous systems.
  - 4.1 Dynamical systems. Critical points and periodic orbits. Stability. Conjugation of dynamical systems.
  - 4.2 Tubular flow theorem. Hartman's theorem.
  - 4.3 Qualitative study of the autonomous linear equation.

## Methodology

Fundamental in the learning process of the subject is the work by the student, who can count on the guidance of the teacher at each moment.

There will be three types of guided activities:

*Theory Classes:* The teacher introduces the basic concepts of the subject matter showing examples, demonstrating properties and fundamental results. The student must complement the teacher's explanations with personal study.

*Classes of Problems:* We work on the understanding and application of the concepts and tools introduced to theory, with the realization of theoretical and/or practical exercises. It is well known that the only way to learn mathematics is by solving lots and lots of problems. For this reason we think that the student must dedicate a minimum of 5 hours a week to solving problems in this subject. The student will have a list of problems for each theme, which he must think about, try to solve and which will be worked on in the problem classes. A delivery of problems is requested for each theme to ensure that this work is done continuously.

*Computer practices:* in each session a different type of differential equation will be dealt with to model a real situation and predict future behaviors depending on circumstantial parameters.

The exercises that appear in the lists of Problems or Computer Practices and that have not finished in the corresponding session will have to solve them the student like part of his autonomous work.

The notes on the Theory, the lists of Problems and Computer Practices will be posted on the subject's Moodle Aules website; a summary of the Theory and Problem classes is also posted weekly.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Practical modelization problems	12	0.48	
Theory classes	30	1.2	
Type: Supervised			
Problem classes	30	1.2	
Type: Autonomous			
Study of the theory and resolution of problems	114	4.56	

## Assessment

50% of the course will be evaluated in a continuous way.

Continuous evaluation:

- Control at the end of 2-3 practical sessions (20%) and 1 final exam of practical sessions (10%). Denote its score by NotaPrt/10
- 2 handouts of Problems (10%). Denote their scores by LI1/10 and LL2/10 respectively.
- 2 partial exams, each counting for 30% in the overall note. Denote their scores by EP1/10 and EP2/10 respectively.

Evaluation that can be repeated to get a new opportunity:

- In case one takes the opportunity to improve the actual score, this new opportunity counts for 50%. No mention of honors is made when taking the new opportunity.

To pass the course it is necessary to have obtained an average note of at least 35% for the partial exams (resp. new opportunity)

More concrete, we apply the following averaging to obtain the final note for the course:

- If  $(EP1+EP2)/2 < 3.5$ , then the score before taking a new opportunity is  $NF1 = (EP1+EP2)/2$ .
- If  $(EP1+EP2)/2 \geq 3.5$ , then the score before taking a new opportunity is  $NF1 = 0.05*(L1+LL2) + 0.3*NotaPrt + 0.3*(EP1+EP2)$ .

In each case, when going for a second opportunity, obtaining the score ER/10 for this exam, then the final and definite score NF2 will be:

- If  $ER \geq 3.5$ :  $NF2 = 0.05*(L1+LL2) + 0.3*NotaPrt + 0.05*(EP1+EP2) + 0.5 ER$
- If  $ER < 3.5$ :  $NF2 = ER$

## Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Exam of practical sessions	10%	3	0.12	5, 4, 3, 6
Handout of practical work	20%	12	0.48	5, 4, 3, 6, 7
Handout of problemes	10%	12	0.48	2, 1, 4, 3, 6, 7
Partial exams	60%	8	0.32	2, 1, 4, 3, 6
Repeated final exam	50%	4	0.16	2, 1, 4, 3, 6

## Bibliography

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## Software

In the computer sessions the students makes use of the programs SAGE and Excel; to resolve the problems proposed in these sessions students often receive information in the language of the computer algebra package of Mathematica.

There will also made use of the program P4 to show the behavior in the neighbourhood of critical points for polinomial systems in two dimensions.