

**Differential equations and modelling II**

Code: 100101  
ECTS Credits: 6

| Degree              | Type | Year | Semester |
|---------------------|------|------|----------|
| 2500149 Mathematics | OB   | 3    | 2        |

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

## Contact

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## Use of Languages

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: Yes  
Some groups entirely in Spanish: No

## Teachers

Salvador Borrós Cullell

## Prerequisites

Mathematical analysis in one and several variables, Linear Algebra and a first course on Differential Equations and modeling.

## Objectives and Contextualisation

This subject is the second part of a two semesters course of introduction to ordinary differential equations (ODE). It has both theoretical and applied sides. It is aimed that the students know and are able to apply the concepts of the qualitative theory of ordinary planar differential equations and also that they have a basic knowledge of the paradigmatic partial differential equations. During this semester we will apply many of the results established and studied in the first course on ODE and at the same time we will introduce new tools for studying the mentioned differentiated equations.

## Competences

- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.

## Learning Outcomes

1. Extract quantitative information about the solution to an ordinary differential equation, without the need to resolve it.
2. Know how to draw simple phase portraits of systems of planar differential equations.
3. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
4. Study the behaviour of the solutions to differential equation systems in accordance with the parameters defining them.

## Content

This subject is structured in two parts. The first one is about the qualitative theory of ordinary differential equations, with special emphasis on planar autonomous systems. It is an introduction of which later can be studied in more depth in the course "Dynamical systems". The second is a first study of the most famous partial differential equations and also has continuity in the course "Partial differential equations".

### 3.1 Autonomous systems in the plane.

3.1.1. Autonomous systems in  $\mathbb{R}^n$ . Geometric interpretation. Structure of the orbits. First integrals. Invariant surfaces. Phase portraits and conjugation.

3.1.2. Integrable systems. Phase portrait of planar integrable systems: potential systems, Hamiltonian systems, the model of Lotka-Volterra.

3.1.3. Non-integrable systems: flow box theorem, qualitative analysis of equilibrium points, limit behavior of the orbits, Bendixson-Poincaré theorem, Lyapunov functions. Limit cycles. Criterion of Bendixon-Dulac. Models of ecology. Van der Pol system.

### 3.2 First order partial differential equations.

3.2.1. Introduction to partial differential equations(PDE).

3.2.2. Linear and quasi-linear PDE of first order.

### 3.3 Second order partial differential equations.

3.3.1. The wave equation on an infinite string. D'Alembert's formula. Boundary value problems.

3.3.2. The heat equation. The case of an finite bar.

3.3.3. Separation of variables and Fourier series.

3.3.4. Laplace's equation.

## Methodology

The objective of the classes of theory, problems and practices is to give students the most basic knowledge of the

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Activities

| Title | Hours | ECTS | Learning Outcomes |
|-------|-------|------|-------------------|
|-------|-------|------|-------------------|

Type: Directed

|                     |    |      |
|---------------------|----|------|
| Classes of problems | 15 | 0.6  |
| Classes of theory   | 30 | 1.2  |
| Type: Supervised    |    |      |
| Practical classes   | 6  | 0.24 |
| Type: Autonomous    |    |      |
| Personal studies    | 88 | 3.52 |

## Assessment

There will be two partial exams during the course, one in the middle of the course and the other at the end. Also the students must solve a problem with the

help of the computer. If the assesement of the first partial is EP1, of the second partial is EP2, and the assesement of the pratical exercise is P, the final note

will be  $F=(4EP1+4EP2+2P)/10$ , but for pass this course it is necessary that  $P \geq 4$ .

If  $F < 5$  the student will have a final exam (FE), the assesement of this final exam is  $(8FE+2P)/10$ , again for pass this course it is necessary that  $P \geq 4$ .

## Assessment Activities

| Title               | Weighting | Hours | ECTS | Learning Outcomes |
|---------------------|-----------|-------|------|-------------------|
| Final test          | 80%       | 4     | 0.16 | 4, 1, 3, 2        |
| First test          | 40%       | 3     | 0.12 | 4, 1, 3, 2        |
| Practical exercices | 20%       | 0     | 0    | 4, 1, 3, 2        |
| Second test         | 40%       | 4     | 0.16 | 4, 1, 3, 2        |

## Bibliography

The basic books for the first part of the course are:

"Ecuaciones diferenciales, sistemas dinámicos y álgebra lineal", Morris W. Hirsch, Stephen Smale, Alianza Universidad Textos, Madrid, 1983.

."Equações Diferenciais Ordinarias", J. Sotomayor.

"Qualitative Theory of Planar Differential Systems", Freddy Dmortier, Jaume Llibre Joan C. Artés, Universitext, Springer, 2006.

For the second part:

"Primer curso de ECUACIONES EN DERIVADAS PARCIALES", Ireneo Peral, UAM, Madrid, 1995. (pdf accessible a la plan web del professor)

"EDP, um curso de graduação", Valéria Lório, IMPA, Brasil, 2001.

Complementary books:

"Models amb Equacions Diferencials", R. Martínez. Materials de la UAB no. 149. Bellaterra, 2004

"Equações Diferenciais: Teoria Qualitativa", L. Barreira i C. Valls, IST Press Lisboa 2010.

"Ecuaciones Diferenciales y Cálculo Variacional ", Lev Elsgoltz, Mir, Moscou, 1983.

"Apunts d'Equacions Diferencials", d'en Francesc Mañosas, UAB (accessible via el Campus Virtual)

"Ecuaciones diferenciales", V. Jimenez. Serie: enseñanza. Universidad de Murcia, 2000.

Análise de Fourier e equações diferenciais parciais", Djaro guedes de Figueiredo, IMPA, Brasil, 2000.

"Càlcul Infinitesimal amb Mètodes Numèrics i Aplicacions", C. Perelló. Enciclopèdia Catalana, 1994.

"Ecuaciones Diferenciales y Problemas con Valores en la Frontera" „E. Boyce, y R.C. Di Prima, Ed. Limusa, México, 1967.

"Partial Differential Equations", Lawrence C. Evans, GSM 19, AMS, Providence, 1991.

"Partial Differential Equations, An Introduction", Walter Strauss, Wiley, New York, 1992.

## **Software**

20 hours of theory classes, 15 hours of problem classes, 6 hours of practice classes