2021/2022

Galois theory

Code: 100102
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	1

The proposed teaching and assessment methodology that appear in the guide may be subject to changes as a result of the restrictions to face-to-face class attendance imposed by the health authorities.

Contact

Name: Francesc Xavier Xarles Ribas

Email: Xavier.Xarles@uab.cat

Use of Languages

Principal working language: catalan (cat)

Some groups entirely in English: No

Some groups entirely in Catalan: Yes

Some groups entirely in Spanish: No

Other comments on languages

This document is an unsupervised translation. In case of discrepancy, the Catalan version shall prevail.

Teachers

Pere Ara Bertrán

Prerequisites

Background on Group Theory is required (e.g. the notions introduced and studied in "Estructures Algebraiques"). Groups are used in an essential way in this module. Thus, in order to be able to work with concrete examples, it is particularly interesting to have some familiarity with groups of small order.

It is also important to be familiar with basic notions of Ring Theory (again, the ones introduced in "Estructures Algebraiques). Of particular importance are the notions related to irreducible polynomials, as well as the theory of finite fields.

Objectives and Contextualisation

The main objective of this module is to develop the notions of Galois Theory and their applications to problems related to resolubility of equations. The latter problems arise as some of the oldest in the History of Mathemathics. Their roots can be traced back to the Babylonia era and culminates brilliantly with the work of Évarist Galois, whose work develops the theory of solvability by radicals.

The modern approach to Galois Theory constitutes a central theme in Algebra, since the abstract methods used show the power of (previously introducted) tools in action. Thus, the translation of a problem to Field Theory, and subsequently to Group Theory (and back) show how abstract, seemingly different branches of of Mathematics interact to solve a classical, more applied problem.

We will start introducing the problem of solving an equation by radicals in its historical context. Next, Field Theory will provide the formal framework where to formulate the problem and study effectively the Galois Theory of equations.

A fundamental tool here is provided by the techniques coming from Group Theory, particularly when it comes to examples and manipulation. However, due to time constraints, we shall review only the most basic concepts and refer to the notions studied in the course "Estructures Algebraiques".

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of ones work.
2. Calculate groups of low degree Galois equations and deduce their resolvability by radicals.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Manipulate expressions involving algebraic and transcendent elements.
5. Relate geometric constructions with algebraic extensions.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Basic Notions

Polynomial equations: the formulas for small degree.

Rings, ring homomorphisms. The field of fractions of a commutative domain.

Subrings and subfields generated by elements.

The characteristic of a field.

2. Field extensions

Algebraic and transcendental elements.

The degree of a field extension. The multiplicativity formula for degrees.

Algebraic extensions.

Extensions de homomorphisms. The group Gal(L/K).

The spitting field of a polynomial.

Finite fields.

3. Normal and separable extensions

Normal extensions.

Formal derivative of a polynomial and polynomials of multiple roots.

Separable elements and separable extensions.

4. The Fundamental Theorem of finite Galois Theory

Galois extensions. Artin's Theorem.
Galois correspondence: the fundamental theorem.

5. Galois theory of equations.

Solvability by radicals and solvable groups.

Cyclotomic and cyclic extensions.

Galois Solvability Theorem .

Polynomials whose Galois group is S_p, where p is prime.

6. Fundamental theorem of algebra.

Methodology

There will be two lectures and one tutorial per week, during 15 weeks. In addition, there will be 3 seminar sessions of 2 hours each, distributed in the semester. Students are strongly encouraged to attend lectures, tutorials, and seminars.

During the lectures, the main tools needed for understanding the subjet and also for problem solving will be introduced. Problem solving will be the main focus in the tutorials, where also a better understanding of the concepts introduced in the lectures will be achieved. Students participation in the form of discussion will be part of the methodology.

In seminars, students participation will be more prominent as these are designed in the form of hands-on exercises and focusing, in particular, in manipulation of examples.

Various resources will be offered through moodle. In particular, problems/seminars and additional material that may complement the subject matter of the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	30	1.2	2, 3, 1, 4, 9, 8, 7, 6, 5
Seminars	6	0.24	2, 3, 1, 4, 9, 8, 7, 6, 5
Tutorials	15	0.6	2, 3, 1, 4, 9, 8, 7, 6, 5
Type: Autonomous			
Course work (from lectures)	27	1.08	2, 4, 9, 8, 6, 5
Exams preparation	16	0.64	2, 4, 8, 6, 5
Problem solving	40	1.6	2, 3, 4, 8, 6, 5
Seminar preparation	10	0.4	2, 3, 4, 8, 7, 6, 5

Assessment

The distribution of marks will be done as follows:

35% of the grade corresponds to the Intersemester exam.

15% of the grade corresponds to problem and/or practical assignments.

50% of the grade corresponds to the exam at the end of semester.

There will be a resit exam, that will substitute the grade corresponding to the Intersemester exam and the exam taken at the end of semester. To be able to do this exam, the student must have done both exams and to have a grade inferior to 5.

Qualification of Non-Assessed. A student will be classified as non-assessed if the following conditions are met:
He/she does not turn up to the exam realized in January-February,
he/she does not turn up to the resit exam in February.

Higher academic distinction (Honors): After the exam carried out at the end of the semester, the higher academic distinctions will be awarded.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Exam	50%	3	0.12	2, 3, 1, 4, 9, 8, 6, 5
Intersemester exam	35%	2	0.08	2, 3, 1, 4, 9, 8, 6, 5

Problem assignments	15%	1	0.04	2, 1, 4, 9, 8, 7, 6, 5

Bibliography

F.Bars, Teoria de Galois en 30 hores, http://mat.uab.cat/~francesc/docencia2.html

David A. Cox, Galois Theory. Hoboken : Wiley-Interscience, cop. 2004
http://syndetics.com/index.aspx?isbn=0471434191/summary.html&client=autbaru&type=rn12

D. S. Dummit, M. R. Foote, Abstract Algebra, Wiley, 2004.

D.J.H. Garling. A course in Galois Theory. Cambridge Univ. Press, 1986.

J. Milne. Fields and Galois Theory, http://www.jmilne.org/math/

P. Morandi. Fields and Galois Theory. GTM 167, Springer.

S. Roman. Field Theory. GTM 158, Springer.

Ian Steward "Galois Theory" Chapman & Hall / CRC, 2004
http://syndetics.com/index.aspx?isbn=1584883936/summary.html&client=autbaru&type=rn12

Additiona bibliography:

Michael Artin, "Algebra" Prentice Hall, cop. 2011
http://syndetics.com/index.aspx?isbn=9780132413770/summary.html&client=autbaru&type=rn12

T. Hungerford, "Algebra" New York : Springer-Verlag, cop. 1974
http://syndetics.com/index.aspx?isbn=0387905189/summary.html&client=autbaru&type=rn12

Jean-Perre Tignol, "Galois' Theory of Algebraic Equations". World Scientific 2001

A. M. de Viola Priori, J.E. Viola-Priori. Teoría de cuerpos y Teoría de Galois. Reverté (2006).

Galois' life (novel, in Catalan):

Josep Pla i Carrera. Damunt les espatlles de gegants. 1ra Edició: Editorial la Magrana. 2na Edició: Edicions FME http://www.fme.upc.edu/ca/arxius/damunt-les-espatlles-dels-gegants_jpla

Some interesting links:

http://www.galois-group.net
http://godel.ph.utexas.edu/~tonyr/galois.html
htpp://www-groups.dcs.st-andrews.ac.uk/%7Ehistory/Mathematicians/Galois.html
Curiosinats origami: Robert J. Lang: http://www.langorigami.com
Tom Hull: http://www.merrimack.edu/thull/~omfiles/geoconst.html
Koshiro Hatori: http://origami.ousaan.com/library/conste.html
http://www.langorigami.com/science/mathlinks/mathlinks.php4.

Software

We will use SageMath.