Universitat Autònoma de Barcelona

2022/2023

Calculus of several variables and optimization

Code: 100093
ECTS Credits: 9

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	1

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Joan Josep Carmona Domènech
Ignasi Guillén Mola
Laura Prat Baiget

Prerequisites

The main prerequisite is a standard first-year course in Calculus, covering differential and integral calculus in one real variable. If not fulfilled, it is imperative that, at least, the student understands the notion of convergence of functions or sequences, as well as those of continuity, derivability and integrability of functions. It is also very important that the student is familiar with the computation of limits, differentiation rules, the fundamental theorem of calculus, Taylor's development of elementary functions, etc.

Objectives and Contextualisation

The main objective of this course is to know the basic techniques of the differential and integral calculus in several variables and the most important concepts of vector analysis.

In the first part of the course the student must first become acquainted with the Euclidean space and its metric and topological structure. Next, the key concept is that of differential as a linear approximation of the increment and other better approximations, in terms of higher order differentials, and how the behavior of these approximations translates into local properties of the function. In the same way that in the case of one variable, the techniques of the course will be applied to the resolution of different mathematical problems such as geometric problems, optimization or, in general, issues where quantify the variation of a given magnitude based on other variables. The student must also be familiar with the geometric concepts of curves and regular surfaces, tangential planes, local coordinates, etc.

The second part of the course, more instrumental, is dedicated to multiple integrals and to vector analysis, covering techniques such as change of order of integration, change of variables, Green, Gauss or Stokes's theorems.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Contrast acquired theoretical and practical knowledge.
4. Know how to apply the theorems of Inverse Function and of the implicit function to specific problems.
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
7. Understand the basic results of Differential Calculus in different real variables.
8. Use algebraic tools in different fields.

Content

1. Differential calculus in several variables:

- Basic geometrical and topological notions in the Euclidean space. Limits and continuity. Parameterization. Graphics and level sets

- Differentiability. Basic properties. Partial derivatives and directional derivatives Relative extremes

- Higher order differentials. Taylor's Formula. Analysis of the critical points.

- The inverse function theorem. Changes of coordinates.

- The implicit function theorem. Geometric viewpoint, regular submanifolds.

- Functional dependence and independence.

- Constrained extrema. Lagrange multipliers

2. Integration

- The Riemann integral of bounded functions on rectangles. Basic properties

- Fubini theorem.

- Integration on general sets

- Changes of variables, meaning of the Jacobian.

- Length and area. Integration on curves and surfaces.

3. Vector analysis

- Orientable surfaces.

- Circulation and flow of a vector field.

- Divergence and rotation of a vector field. Conservative and solenoidal fields.

- Theorems of Green, Gauss and Stokes.

Methodology

There are three type of activities the student is supposed to attend: the lectures (3 hours /week) mainly concerned with the description of the theoretical concepts, problem solving sessions (1 hour/week) and seminars (2 hours on alternate weeks), similar to the problem solving sessions but where students work in groups supervised by a teaching assistant.

The course has a web page in the UAB online campus gathering all information and communications between students and professors, and where all material, including problem sheets, solutions, etc are published regularly.

Students are supposed to submit a couple of exercise sets to be evaluated in a personalized interview.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	39	1.56	
Problem session	13	0.52	
Working seminars	13	0.52	
Type: Autonomous			
Solving problems	95	3.8	

Studying theoretical concepts	53	2.12

Assessment

A continuous assessment is done consisting in a mid-term test (P) and two evaluable seminars (LL1,LL2), mandatory. At the end there is a final term test (F) and a resit exam (R)

The final score is obtained in two steps. Let P,LL,F,R denote respectively the scores (between 0 and 10) of the mid-term test, the mean of the two seminars, the final and the resit exams.

With P,F we compute NE=(0,5)F+(0,5)P and then C1=(0,80)NE+(0,20)LL. If C1 is greater or equal to 5, C1 is the final score. Otherwise, the students that have submitted the two exercise sets may attend the resit exam. Then the final score is C2=(0,80) R+(0,20)LL.

Students with C1 greater or equal to 5 may attend R to improve their grades, in which case the final score is (C1+C2)/2.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final term Test	40%	4	0.16	7, 6, 4
Mid-term test	40%	4	0.16	7, 6
Seminars	20%	4	0.16	2, 3, 7, 1, 6, 5, 4, 8

Bibliography

- Vector Analysis, J.E. Marsden y A.J.Tromba, Addison Wesley Longman.
- Functions of several variables, Nartin Moskowitz and Fotios Paliogiannis, World Scientific, 2011.
- Analysis II, Terence Tao, Hindustan Book agency, Texts and Readings in Mathematics 38, 2006.
- Functions of several variables, Wendell Fleming, Undergraduate texts in Math, Springer.
- Second year calculus, David Bressoud, Undergraduate texts in Math, Springer, 1991.

Software

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