2022/2023

Complex and Fourier analysis

Code: 100103
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	2

Contact

Name: Joan Josep Carmona Domènech
Email: joanjosep.carmona@uab.cat

Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Juan Jesús Donaire Benito
Juan Carlos Cantero Guardeño

Prerequisites

It is a third-year subject, so students already have some mathematical background needed to follow it. Although it will be quite self-contained some prior knowledge is essential. For example, series theory and power series in Anàlisi Matemàtica and differential calculus in Càlcul en Diverses Variables.

Objectives and Contextualisation

Know and be able to use the fundamental concepts and results of Complex Analysis.

Understand in depth the demonstrations of the most important results and the most common techniques in the area.

Have some initial notions of the basic concepts of the Fourier transform and the Laplace transform.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Recognise the presence of Mathematics in other disciplines.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Comfortably deal with the calculation of residues and its applications
4. Contrast acquired theoretical and practical knowledge.
5. Handle with ease homographic transformations and conformal representation.
6. Know how to calculate Fourier periodic function coefficients and their possible immediate applications for calculating sums of series.
7. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
9. Understand Fourier and Laplace transforms of elemental functions and their application to the resolution of differential equations.
10. Understand the basic results and fundamental properties of holomorphic functions and Cauchy theory.

Content

1. Preliminaries. Complex numbers. Power series. Holomorphic functions. Cauchy-Riemann equations.

2. Cauchy's Local Theory. Complex line integrals. Cauchy-Goursat theorem and the local Cauchy theorem. Holomorphy and analyticity. Zeros of holomorphic functions. The index of a closed curve. Cauchy's integral formula. Analytical extension. Cauchy inequalities, Liouville's theorem and Fundamental theorem of algebra. The principle of the maximum module. Schwarz's motto.

3. Singularities. Laurent series. Classification of isolated singularities. Waste theorem and applications. The principle of argument and Rouché's theorem.

4. Harmonic functions. Basic properties of harmonic functions. Harmonic functions on a disc. Dirichlet problem.

5. Integral transforms. Fourier transform. Laplace transform. Basic properties. Applications to solve equations.

Methodology

The subject has two hours of theory every week. They will be taught traditionally with blackboard. In the theory where the concepts will be shelled and enunciating the important results (theorems) that build the theory that we are introducing.

We will dedicate ourselves to proving theorems and methods of solving by means of examples and exercises.

The student will receive a list of exercises and problems that we will work on in the weekly problem class. Previously, during their house activity, they will have read and thought about the proposed exercises and problems. In this way, their participation in the classroom can be guaranteed and the assimilation of the procedural contents will be facilitated.

There will be three seminar sessions, each of them of two hours. In the first two sessions there will be a first part where the teacher will complement two topics already covered in the theory and problem classes. In the second part, the students will do independently some problem related to what has been explained, it can be done in groups.

The third session of the seminars will be evaluable. If the sanitary conditions allow it will be done in pairs. The topics covered are a more in-depth study of Möbius transformations and further applications of the residue theorem in the calculation of definite integrals. The evaluation will deal with these issues.

In the event that we are forced to teach electronically, sufficient material will be provided for monitoring. The Virtual Campus will be the means of communication between teachers and students. It will be important to consult it on a daily basis.

Students will have a tutoring and advice service both online and in the office. It is recommended to use this help for the follow-up of the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercices	14	0.56	10, 9, 3, 5, 6
Maths seminar	6	0.24	10, 9, 3, 5, 6
Theory	28	1.12	10, 9, 3, 5, 6
Type: Autonomous			
Studying time	88	3.52	10, 9, 3, 5, 6

Assessment

Learning mathematics is a complex process. Maturation is needed that is achieved throughout the course. Many times, some result from the beginning of the theory is understood to be completely advanced in the course. This shows the difficulty of the evaluations.

In the university there is the model of continuous evaluation which is not viable as it is done in secondary education as there is neither the logistics nor the possibilities to carry it out. Then a model is made, which has a certain resemblance to a continuous evaluation, and which forces the students to do the study we can say every day.

There will be two written partial exams during the semester, which will mainly consist of problem solving, but will also contain a theoretical part. They will have a P1 and P2 grade respectively. The first will have a weighting of 35% and the second 45%.

The seminar test will assign an S grade of up to 20%.

The qualification for continuous assessment will be obtained with the formula

$QC = 0.35 * P1 + 0.45 * P2 + 0.2 * S$.

If QC is greater than or equal to 5 the course will be passed. Otherwise, the student will be able to present himself to a recovery, and will obtain a grade R and one has

QC'= 0.8 * R + 0.2 * S.

In order to appear for recovery, the miaximun of P1 and P2 is required to be greater than or equal to 0.5. You can also choose to apply for people who want to improve their grade. The course note will always be

QF=maximum {QC, QC'}.

Possible honors registrations will be awarded in compliance with current regulations and once the entire assessment has been completed, possible recovery included.

If a student has only taken an assessment avaluation they will be given a "No avaluable" final grade.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	35	4	0.16	2, 4, 10, 1, 3, 8, 7
Maths Seminars	20	2	0.08	10, 9, 3, 5, 6
Recovery exam	80	4	0.16	2, 4, 10, 9, 1, 3, 5, 8, 7, 6
Second partial exam	45	4	0.16	10, 9, 3, 8, 7

Bibliography

Bibliografia bàsica:

1) L. Ahlfors, Complex Analysis. Mc Graw-Hill. 3ra edició, 1979.(És una referència clàssica que amb un format reduït tracta moltíssims temes de forma rigorosa).

2) J. Conway, Functions of One Complex Variable, second Edition, Springer Verlag, 1978. (Abarca molt més que el curs i conté molts problemes).

3) J. P. D'Angelo; An introduction to Complex Analysis and Geometry; A.M.S. 2010 (És una introducció de nivell molt més elemental que els anteriors).

4) B. Davis; Transforms and Their Applications, Thrid Edition, Springer (2001) (Serveix com a inici i aprofundiment en l'estudi del món de les transformacions integrals).

Bibliografia complementària:

1) J. Bruna, J. Cufí, Anàlisi Complexa, Manuals UAB 49, 2008.

2) R. Burckel, Introduction to classical complex Analysis, vol I, Academic Pres, 1979.

3) W. Rudin, Análisi Real y Complexo, Alhambra, 1979

4) S. Saks et A. Zygumund, Fonctions Analytiques, Massin et Cie, 1970.

5) M. Stein, R: Shakarchi, Complex Analysis, Princeton University Press, 2003.

Software

There are no computer practice classes in the subject, so no study of computer programs will be done. Despite this, it will be recommended to use mathematical manipulation programs such as Maxima or Wolfram Alpha, which can be very useful.