

**Advanced Mathematical Methods**

Code: 100167  
ECTS Credits: 5

| Degree          | Type | Year | Semester |
|-----------------|------|------|----------|
| 2500097 Physics | OT   | 3    | 1        |

**Contact**

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**Use of Languages**

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: No  
Some groups entirely in Spanish: No

**Other comments on languages**

It could be given in english if the group decides so.

**Teachers**

Santiago Peris Rodriguez

**Prerequisites**

It is advisable to have studied the following subjects:  
Calculus in one variable  
Vector Calculus  
Differential equations

**Objectives and Contextualisation**

This subject introduces some basic mathematical concepts  
needed in physics in general, and in physics / Quantum mechanics  
and field theories, in particular. It is intended that the student  
achieve the understanding of the concepts of Hilbert space, operators, di  
and, especially, groups. It wants to give an integrative vision  
of concepts that appear in different fields in physics. At the same time,  
the student will have to acquire the capacity to apply them with agility  
for different types of problems.

## Competences

- Act with ethical responsibility and respect for fundamental rights and duties, diversity and democratic values.
- Apply fundamental principles to the qualitative and quantitative study of various specific areas in physics
- Be familiar with the bases of certain advanced topics, including current developments on the parameters of physics that one could subsequently develop more fully
- Communicate complex information in an effective, clear and concise manner, either orally, in writing or through ICTs, and before both specialist and general publics
- Develop the capacity for analysis and synthesis that allows the acquisition of knowledge and skills in different fields of physics, and apply to these fields the skills inherent within the degree of physics, contributing innovative and competitive proposals.
- Make changes to methods and processes in the area of knowledge in order to provide innovative responses to society's needs and demands.
- Take account of social, economic and environmental impacts when operating within one's own area of knowledge.
- Use critical reasoning, show analytical skills, correctly use technical language and develop logical arguments
- Use mathematics to describe the physical world, selecting appropriate tools, building appropriate models, interpreting and comparing results critically with experimentation and observation
- Work independently, have personal initiative and self-organisational skills in achieving results, in planning and in executing a project

## Learning Outcomes

1. Classify representations of the most simple groups.
2. Communicate complex information in an effective, clear and concise manner, either orally, in writing or through ICTs, in front of both specialist and general publics.
3. Determine the effect on the observables of symmetry transformation.
4. Determine the observables that characterise representation.
5. Determine the representation that characterizes a particular physical system.
6. Determine the symmetry group (exact or approximate) associated with a physical system.
7. Explain the explicit or implicit code of practice of one's own area of knowledge.
8. Identify situations in which a change or improvement is needed.
9. Identify symmetry groups associated with the laws of physics.
10. Identify symmetry groups associated with theories of fundamental interactions.
11. Identify symmetry groups in addition to their particle and crystallography representations, associated with atomic physics.
12. Identify the social, economic and environmental implications of academic and professional activities within one's own area of knowledge.
13. Obtain representation of simple symmetry groups.
14. Relate continuous groups with the Lie algebra to which they are associated.
15. Relate the symmetries of nature with the appropriate symmetry group (exact or approximate).
16. Use critical reasoning, show analytical skills, correctly use technical language and develop logical arguments
17. Use the tensor calculus.
18. Work independently, take initiative itself, be able to organize to achieve results and to plan and execute a project.

## Content

### PROGRAM

1. Hilbert spaces

- 1.1 Pre-Hilbert spaces.
- 2.2 Hilbert spaces.
- 2. Operators.
- 2.1 Linear operators.
- 2.2 Eigenvalues and eigenvectors.
- 3. Distributions
- 4. Introduction to group theory
- 4.1 Definition and motivation (symmetries)
- 4.2 Examples:  $SO(3)$ ,  $SU(2)$ ,  $SU(N)$  (relation with unitary operators).
- 4.3 Lie algebras (generators of the continuous group)
- 4.4  $su(N)$  (relation with selfadjoint operators) and relation with  $su(2)$  with  $so(3)$
- 5. Representations
- 6. Tensorial methods

## Methodology

This course develops mathematical language and calculation tools that are basic for advanced physics subjects. The personal work of the student is fundamental. Classroom sessions will be divided into:  
 Lectures: The teacher will present the basic concepts and reasoning of the Subject, with the support of examples.  
 Problem classes: Among a collection of problems, the teacher will solve in detail a selection. Students will have to work on their own the

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Activities

| Title  | Hours | ECTS | Learning Outcomes |
|--|-------|------|-------------------|
| Type: Directed   |       |      |                   |
| Blackboard lectures: the professor will expose basic concepts and arguments for each subject, with the support of detailed examples. | 27    | 1.08 |                   |
| Practical lectures: among a problems collection, the professor will solve a set of them. The rest will be solved by the students.    | 14    | 0.56 |                   |
| Type: Autonomous   |       |      |                   |
| Individual and groupal work solving problems   | 28    | 1.12 |                   |

|                                 |    |      |
|---------------------------------|----|------|
| Selective homework              | 11 | 0.44 |
| Study of teoretical foundations | 37 | 1.48 |

## Assessment

Partial examination of Hilbert Spaces and operators: 45% of the note.

Group partial exam: 50% of the mark.

Selective delivery of problems: 5% of the note.

By imperative of the general regulations of the university, to be able to pass

Resit exam of the two partials: 95% of the note. There is no minimum mark

## Assessment Activities

| Title                             | Weighting | Hours | ECTS | Learning Outcomes   |
|-----------------------------------|-----------|-------|------|---|
| Group theory exam                 | 50%       | 2.5   | 0.1  | 1, 2, 6, 4, 3, 5, 7, 9, 10, 11, 12, 8, 13, 16, 14, 15, 18, 17 |
| Hilbert spaces and operators exam | 45%       | 2.25  | 0.09 | 2, 7, 12, 8, 16, 18, 17                                       |
| Homework                          | 5%        | 0.25  | 0.01 | 1, 2, 6, 4, 3, 5, 7, 9, 10, 11, 12, 8, 13, 16, 14, 15, 18, 17 |
| Resit exam                        | 95%       | 3     | 0.12 | 1, 2, 6, 4, 3, 5, 7, 9, 10, 11, 12, 8, 13, 16, 14, 15, 18, 17 |

## Bibliography

Basic bibliografy.

P. Szekeres, *A course in Modern Mathematical Physics*.

Elvira Romera et al., Métodos matemáticos: Problemas de espacios de Hilbert, operadores lineales y espectros

G. Arfken, *Mathematical Methods for Physics*.

Advanced and complementary bibliography.

J.J. Sakurai, *Modern Quantum Mechanics*.

J.F. Cornwell, *Group theory in Physics*.

H. Georgi, *Lie Algebras in particle physics*.

L. Abellanas i A. Galindo, *Espais de Hilbert*.

S.K. Barbarian, *Introducció a l'espai de Hilbert*.

L. Schwartz, *Métodos Matemáticos para las ciencias físicas*.

## **Software**

Not conceived.