

**Basics of Mathematics**

Code: 103301  
ECTS Credits: 7

| Degree                                 | Type | Year | Semester |
|--|------|------|----------|
| 2501922 Nanoscience and Nanotechnology | FB   | 1    | 2        |

**Contact**

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**Use of Languages**

Principal working language: catalan (cat)  
Some groups entirely in English: No  
Some groups entirely in Catalan: Yes  
Some groups entirely in Spanish: No

**Teachers**

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**Prerequisites**

This subject is self-contained in the topics that are treated.

In spite of this it is advisable for the student to have the basic skills with algebraic calculations and basic notions of differential calculus on one variable.

**Objectives and Contextualisation**

(Google translation from Ctalan version)

This subject contains a first introduction to the calculation of complex numbers, and the rest of the subject has basic contents of linear algebra, such as:

- Systems of linear and matrix equations.
- Vectors in  $R^n$ .
- Linear applications.
- Own vectors, their own values and diagonalization
- Applications of diagonalization

**Knowledges**

- Know the complex numbers and their different expressions. To know the operations with the complex numbers, and the roots of the complex numbers.

- Knowing that it is a system of linear equations. Understand the systems resolution methods, namely the Gaussian elimination method. Understand that means discussing a system in which there are several parameters.
- Know that it is an array and what operations can be done between them, paying special attention to the product. Understand the concept of an invertible matrix and its relation to the rank of the matrix. Know how to use the Gauss-Jordan method to calculate the inverse, if any, of an array.
- Know the properties of the calculation of the determinant of a square matrix. Understand the relationship between determinants and invertible matrices. Know how to use the determinants appropriately.
- Understand how to operate with vectors. Knowing that it is a vector subspace of  $R^n$  and in what ways it can be defined.
- Understand the concept of linearly dependent and linearly independent vectors. Knowing that it is a generator system. Interpret the rank in terms of the linear independence of vectors. Understanding the concepts of the dimension of a vector subspace. Understand whether the intersection, the union or the sum of subspaces, is a subspace. Know that they are the components of a vector on a  $R^n$  base and how they vary when changing it.
- Be very clear about the concept of application between arbitrary sets and the different types of applications: injective, exhaustive and bijective. Understand well the concept of application composition and the concept of reverse application.
- Know that given each array it defines a linear application between spaces  $R^n$  and  $R^m$ . Be clear about the definition of the subspace core and image of a linear application and its relation to the injectivity, exhaustivity of the application. Understand the relationship between degrees of freedom of a homogeneous system and the formula of dimensions.
- Understand the parallelism between matrices and linear applications with respect to the product and the composition.
- Know what is an own value and an own vector associated with an endomorphism or a square matrix. Know how to calculate the vaper of its own vectors. Understand that it means that an endomorphism or a square matrix is diagonalized

#### Abilities

- Know how to express a complex number in Cartesian form and in polar form. Know how to operate with complex numbers. Know how to calculate the roots of a complex number.
- Know how to solve a system of linear equations where only numbers appear. Know how to discuss a system of linear equations where parameters appear.
- Have skill in calculation with matrices paying special attention to the product of matrices and the calculation of inverse. Know how to solve a symbolic equation with arrays. Have practice in calculating the rank of a matrix.
- Know how to calculate determinants where numbers and parameters appear, paying more attention to the use of properties than in routine rules.
- Do not have difficulties knowing when some  $v_1, v_2, \dots, v_p$  vectors are linearly dependent. In the case of being linearly dependent know how to find dependency combinations.
- Know how to define a subspace by equations and generator systems and move from one to the other. Know how to find subspace bases that are intersection or sum of others. Know how to change the base.
- Do not have difficulty finding the basis of the kernel and the image of a linear application, although it contains, at most, a parameter in its definition.
- Be able to discuss whether a linear application is injective, or exhaustive or bijective. In case the linear application has a reverse to know how to find it.

- Know how to calculate the own values and subspecies of their own vectors associated with an endomorphism. Know how to discuss whether an endomorphism is diagonalizable or not, and in case it is known to find a diagonal expression and the basic change matrices.
- To be able to solve linear differential equations and systems of first linear differential equations.

## Competences

- Apply the concepts, principles, theories and fundamental facts of nanoscience and nanotechnology to solve problems of a quantitative or qualitative nature in the field of nanoscience and nanotechnology.
- Communicate orally and in writing in one's own language.
- Demonstrate knowledge of the concepts, principles, theories and fundamental facts related with nanoscience and nanotechnology.
- Interpret the data obtained by means of experimental measures, including the use of computer tools, identify and understand their meanings in relation to appropriate chemical, physical or biological theories.
- Learn autonomously.
- Manage the organisation and planning of tasks.
- Obtain, manage, analyse, synthesise and present information, including the use of digital and computerised media.
- Reason in a critical manner
- Resolve problems and make decisions.

## Learning Outcomes

1. Abstract the essential variables of the phenomena studied, relate them to each other and deduce properties.
2. Communicate orally and in writing in one's own language.
3. Correctly use specific computer programs and data processors to accurately determine magnitudes of measurement and estimate the associated uncertainty.
4. Identify the mathematical nature of certain physical and chemical phenomena.
5. Learn autonomously.
6. Manage the organisation and planning of tasks.
7. Mathematize certain physical, chemical or biological processes and use accurate mathematical tools to obtain conclusions and interpret the results.
8. Obtain, manage, analyse, synthesise and present information, including the use of digital and computerised media.
9. Reason in a critical manner
10. Resolve problems and make decisions.
11. Show the necessary calculation skills to work correctly with formulas, chemical equations or physics models.
12. Use graphic and numeric methods to explore, summarise and describe data.

## Content

### 1. Complex numbers

Complex numbers and their properties. Trigonometric and polar forms. Operations with complex numbers. Roots of complex numbers.

### 2. Matrices

Resolution of systems of linear equations. Sum, product, and matrix transposition.

Elemental transformations. Reduction of a matrix to echelon form. Rank of a matrix. Invertible matrices. Determinants.

### 3. Vectors in $\mathbb{R}^n$

Definition and examples. Vector structure of  $\mathbb{R}^n$ . Linear dependence and independence. Vector subspaces and generating systems. Basis, coordinates, and dimension. Basis of the intersection and the sum of subspaces. Change of basis matrices.

### 4. Linear maps

Definition and examples. Matrix representation. Composition. Dependence of the matrix with respect to changes of basis. Kernel, image, and rank. Calculation of basis of the subspaces kernel and image.

### 5. Diagonalisation

Eigenvectors and eigenvalues of an endomorphism. Characteristic polynomial. Diagonalisation criteria.

### 6. Applications of diagonalisation

Sequences with linear recurrences. Linear differential equations and systems of first order linear differential equations.

## Methodology

(Google translation from Catalan version)

The subject consists of three main activities.

Theoretical classes in which the concepts and scientific and technical knowledge specific to the subject are introduced and desiccated. and necessary to solve problems.

Classes of problems, complementary to theory classes. In these exercises will be solved and will be deepened in the understanding of the new concepts and scientific and technical knowledge exposed in the theory classes. Normally the student thinks and tries to solve the problems that the classes are discussed and the final optimal solution is reached.

Finally, there will be 2 practice sessions in the computer room, where specific software for mathematical calculations such as Maxima or Sage will be used.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Activities

| Title            | Hours | ECTS | Learning Outcomes |
|------------------|-------|------|-------------------|
| Type: Directed   |       |      |                   |
| Computer room    | 4     | 0.16 |                   |
| Problem sessions | 15    | 0.6  |                   |
| Theory classes   | 45    | 1.8  |                   |
| Type: Supervised |       |      |                   |
| Tutorials        | 6     | 0.24 |                   |
| Type: Autonomous |       |      |                   |

|                    |    |      |
|--------------------|----|------|
| Problem resolution | 48 | 1.92 |
| Study              | 48 | 1.92 |

## Assessment

There are two written exams, a mid-term exam approximately at half semester with a weight of 35% of the final grade of the subject and a final exam with a weight of 50%.

The practices will be evaluated and represent the remaining 15% of the final grade of the course.

Students who have taken both written exams but have not obtained a final grade greater than or equal to 5 out of 10, may opt for reassessment. The reassessment consists of a comprehensive exam of the subject. If the weighted average of this exam, with a weight of 85%, and the mark of practices, with a weight of 15%, is greater than or equal to 5 out of 10, the subject will be considered passed with 5.0. Otherwise it will be considered failed with the average mark obtained.

The Honors qualification (MH) is a decision of the teaching staff responsible for the subject. The UAB regulations indicate that MH can only be awarded to students who have obtained a final grade of at least 9.0 out of 10.0. It can be granted to up to a 5% of the total number of students enrolled.

A student will be considered non-evaluable (NA) if she or he does not make at least 50% of the activities of evaluation of the subject.

The dates of examinations and practical assessments as well as other relevant information or dates that occur throughout the course will be communicated via the virtual campus. It is understood that this is the usual platform for exchanging information between teachers and students.

## Assessment Activities

| Title                   | Weighting | Hours | ECTS | Learning Outcomes               |
|-------------------------|-----------|-------|------|---------------------------------|
| Evaluation of practices | 15%       | 2     | 0.08 | 2, 11, 8, 9, 3                  |
| Mid-term exam           | 35%       | 3     | 0.12 | 1, 5, 2, 11, 6, 4, 7, 9, 10, 12 |
| final exam              | 50%       | 4     | 0.16 | 1, 5, 2, 11, 6, 4, 7, 9, 10, 12 |

## Bibliography

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E. Nart X. Xarles, Apunts d'àlgebra lineal, Materials de la UAB, núm. 237, 1a edició.

D.C. Lay, Álgebra lineal y sus aplicaciones, Pearson Educación, 2016 (ebook)

Grossman, Stanley I., *Álgebra lineal*. Mc Graw Hill, 2012, 7a edició. (eBook)

## Software

Python