Universitat Autònoma de Barcelona

## Functions of a Real Variable

Code: 100087
ECTS Credits: 12

| Degree | Type | Year | Semester |
| :--- | :--- | :--- | :--- |
| 2500149 Mathematics | FB | 1 | A |

## Contact

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Teaching groups languages
You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

## Teachers

Arturo Nicolau Nos
Juan Eugenio Mateu Bennassar
Marc Magaña Centelles

## Prerequisites

It is essential that the students are able to handle with algebraic manipulation of fractions, expressions that contain roots and powers, resolution of linear systems and basic arithmetic of numbers and polynomials. It is also recommended to know the elementary properties of trigonometrical functions. Finally, we hope that the student can do, without much difficulty, the graphic representation of relatively simple functions of one variable. We also presume that the person who attends this course is familiar with logical reasoning and who knows how to deny sentences or proposals.

The most important requirement is, however, a great curiosity to understand and deepen the concepts that will be studied.

## Objectives and Contextualisation

The objective of the subject is that the student learns solidly the basic concepts of the Infinitesimal Calculus: functions of discrete variable (sequences) or continuous, the concepts of limit, derivative and the theory of integration. It is also a basic objective to achieve a certain skill in the manipulation and calculation of limits, derivatives and integrals and to know how to apply the fundamental theorems of this theory. Finally, there is also a generic educational objective: that the student begin to develop the ability to analyze and to reason rigorously.

## Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.


## Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Assimilate the mathematical concepts and objects of the subject as described in its contents.
4. Calculate and study the extrema of functions.
5. Calculate derivatives of functions using the chain rule, the implicit function theorem, etc.
6. Calculate integrals of functions of a variable.
7. Contrast, if possible, the use of calculus with the use of abstraction in order to solve a problem. Evaluate the advantages and disadvantages of the two methods.
8. Deal with inequalities and successions, analyse and draw functions, deduce the properties of a function on the basis of the graph, understand and work intuitively, geometrically and formally with the notions of limit, derivative and integral.
9. Demonstrate knowledge of the inherent objects of calculus with functions of a real variable and of its properties and uses.
10. Develop autonomous strategies for problem-solving such as identifying the field of course-related problems, discriminating routine problems from non-routine ones, designing an a priori strategy to solve a problem, evaluating a strategy and modifying it if necessary and in the context of a specific problem translate theoretical results that could be useful for resolving it.
11. Draft short, orderly and accurate mathematical texts (exercises, resolution of theoretical questions, etc.)
12. Follow and understand an oral explanation of a mathematical subject related with the course.
13. Know how to explain ideas and concepts from the course, and know how to communicate one's own reasoning to third parties.
14. Read and understand course level mathematical texts.
15. Solve problems that imply the use of integrals (longitudes, areas, volumes, etc.).
16. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

## Content

I. The real line.
II. Sequences of real numbers.

- Sequences. Limit. Monotonicity
- Accumulation points. Subsequences.
- The Bolzano-Weierstrass theorem
- Cauchy sequences.
III. Continuïty of one variable functions.
- Functions of one variable. Limits.
- Continuïty.
- Bolzano and Wierstrass theorems
- Monotone functions. Inverse function.
IV. Differential Calculus.
- Derivative of a function at a point. Derivative of a function. Algebraic properties.
- The chain's rule. Derivative of the inverse.
- Critical points, maxima and minima.
- Rolle's theorem. Mean value theorem. L'Hopital's rule.
V. Aproximation by Taylor's polinomial
- Contact order between fonctions
- Taylor polynomial. Properties
- Local study of a function.
VI. Riemann integral
- Computing areas. Lower and upper sums. Integrability. Integral
- Fundamental Theorem of Calculus
- Calculation of primitives
- Geometric and phisic applications of the integral


## Methodology

The subject has one group of theory, two problem groups and four seminars-practicals. The group to which the student belongs can be consulted on the website of the degree course http://mat.uab.cat/gmat.
There will be two sessions of one hour a week of theory and two sessions of problems. This time distribution may be affected by measures against COVID. The Seminars will be devoted to work in a tutored group. The hours and classrooms must be consulted on the website of the degree. In the Moodle of the subject, the student will have at his disposal the necessary material to follow all the sessions There you can find, notes, lists of problems, observations made by teachers or teacheews that may be relevant to the development of the subject.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Activities

| Title | Hours | ECTS |
| :--- | :---: | :--- |
| Type: Directed |  | Learning Outcomes |
| Theory classes | 59 | 2.36 |

Type: Supervised

| Classes of problems | 30 | 1.2 |
| :--- | :--- | :--- |
| Tutored activities | 25 | 1 |
| Type: Autonomous | 100 | 4 |
| Realization of problems | 22 | 0.88 |
| Test Preparation | 50 | 2 |
| Theory Study |  |  |

## Assessment

(Translated with Google Translator. The official version is the catalan one)

The subject has a single call that closes in July.

There will be two short tests, one per semester, which will provide a T grade.
Some seminar sessions will be assessable. These tests will result in an S grade.

There will be two partial tests at the end of each semester with grades P1, P2.

Based on these activities, a Final evaluation grade will be obtained, given by

Final = 0.2 T + 0.15 S + 0.3 P1 + 0.35 P2

If the final grade is greater than or equal to 5 , the student has passed the subject. Students who have not passed the subject will be able to take a final recovery test where they can recover $85 \%$ of the grade.

One dayassessment.

Students who have requested it can use the single assessment modality (see the Faculty's website). The single assessment implies the irrevocable waiver of the right to continuous evaluation.

The student who chooses this mode of assessment will, on the date of the second term, take three tests: an oral theory test, a written problem test and a written test corresponding to the contents of the seminars. The weight corresponding to each part is $25 \%$ theory, $60 \%$ problems and $15 \%$ seminars.

If the student does not pass the subject, he/she can opt for the make-up exam under the same terms as the rest of the students.

## Assessment Activities

| Title | Weighting | Hours | ECTS | Learning Outcomes |
| :--- | :--- | :--- | :--- | :--- |
| Continuous evaluation | 30 | 10 | 0.4 |  |
| Semester exam February | 30 | 2 | 0.08 | $2,3,5,4,7,1,9,10,14,8,16,11,13,12$ |
| Semester exam June | 35 | 2 | 0.08 | $2,3,5,4,6,7,1,9,10,14,8,16,11,15,13,12$ |

## Bibliography

M. Spivak. Calculus. Càlcul Infinitesimal. Ed. Reverté, Barcelona 1995.
F. Mañosas Apunts de Funcions de variable Real. Campus Virtual
R. Larson, R. P. Hostetler, B. Edwards. Cálculo I. Ediciones Pirámide. 2002.
J. M. Ortega. Introducció a l'Anàlisi Matemàtica. Manuals de la Universitat Autònoma de Barcelona 4, Bellaterra 1990.
W. Rudin. Principios de Análisis Matemático. Ed. McGraw-Hill. 1980.

## Software

The use in this course of any special software or other informatic resouces is not under scope

