Universitat Autònoma de Barcelona

2023/2024

Differential equations and modelling II

Code: 100101
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	2

Contact

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Teaching groups languages

You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Teachers

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Prerequisites

Mathematical analysis in one and several variables, Linear Algebra and a first course on Differential Equations and modeling.

Objectives and Contextualisation

This assignment is the second part of a course on fundamental differential equations. Similar to the assignment Differential Equations and Modeling I, this second part has both a theoretical (which will be explored in theory and problem-solving classes) and an applied (which will be introduced in theory classes and applied both in problem-solving and practice classes) components. It is important that the students understand how to apply the concepts of the quantitative theory of differential ordinary equations to problems and have a knowledge of the basic partial differential equations. In addition to applying a number of the well-established and well explored findings from Equations Differential and Modeling I, new techniques for investigating the named differential equations will also be introduced.

Competences

- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.

- Formulate hypotheses and devise strategies to confirm or reject them.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.

Learning Outcomes

1. Extract quantitative information about the solution to an ordinary differential equation, without the need to resolve it.
2. Know how to draw simple phase portraits of systems of planar differential equations.
3. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
4. Study the behaviour of the solutions to differential equation systems in accordance with the parameters defining them.

Content

There are three sections to this topic. The first one focuses specifically on planar autonomous systems and discusses the qualitative theory of ordinary differential equations. It serves as an introduction to a topic that will later be covered in greater detail in the *Dynamical systems course*. The second and third one, have continuity with the course *Partial differential equations* and are a first study of the most well-known partial differential equations.

1 Autonomous systems in the plane.

1.1. Autonomous systems in R^n. Geometric interpretation. Orbits' structure. First integrals. Invariant surfaces. Phase portraits and conjugation.

1.2. Integrable systems. Phase portrait of planar integrable systems: potential systems, Hamiltonian systems, the model of Lotka-Volterra.

1.3. Non-integrable systems: flow box theorem, qualitative analysis of equilibrium points, limit behavior of the orbits, Bendixson-Poincaré theorem, Lyapunov functions. Limit cycles. Criterion of Bendixon-Dulac. Models of ecology. Van der Pol system.

2 First order partial differential equations.

2.1. Introduction to partial differential equations(PDE).

2.2. Linear and quasi-linear PDE of first order.

3 Second order partial differential equations.

3.1. The wave equation on an infinite string. D'Alembert's formula. Boundary value problems.

3.2. The heat equation. The case of a finite bar.

3.3. Variable separation and Fourier series.

3.4. The Laplace's equation.

Methodology

There will be three different kinds of interactive activities: theoretical classes, problem-solving classes, and practical classes.

- The teacher will use the theoretical classes to motivate the class with the study subjects, explain the material, and incorporate motivating examples.
- The solutions to a few representative problems will be discussed in problem-solving classes, and students' active participation will be encouraged. Problem lists may contain more problems than those that will be resolved in class.
- The three course topics will be examined in greater detail in the practical classes. Each student will be required to deliver some problems related to the same (the delivery will be after the day of practice).

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Classes of problems	15	0.6	
Classes of theory	30	1.2	
Type: Supervised			
Practical classes	6	0.24	
Type: Autonomous			
Personal studies	88	3.52	

Assessment

Continuous assessment /Ongoing evaluation:

•A mandatory delivery of practices. We referred to the note on 10 received with the deliveries as *PR*. This is a non-recoverable activity.

• A partial first exam halfway through the theory and problems course. We refer to the note on 10 as *E1*.

• A second partial exam on theory and problems at the end of the course. We refer to the note on 10 as *E2*.

• In the theoretical classes, there may be two optional assignments that are added to E1 and/or E2, respectively, and have a maximum point value of 0.5 before the first and second partial exams. It is necessary for the student to receive a grade higher than or equal to 3.5 in the respective partial in order to add this qualification.

Unique assessment:

- A student who has requested a single evaluation method is required to take a final test on the same day as their peers who are taking the second part of the exam. This test will be a single examination (*SE*)

that includes all of the E1 and E2 evaluation criteria and is weighted at 10. The practice reports that were provided to him/her on the virtual campus will be turned in when the exam is over, and this delivery's note will be qualified with *PR*. As with the other students, it is essential that PR>=4 and recovery is NOT possible. In the same way, SE must be greater than 3.5, and if it falls short, the student has the chance to pass SE by passing a comprehensive exam covering the entire course (see the next paragraph talking about this).

Recovering exam for both cases above:

- Exam covering the entire course with a grade of FE, also about 10. In no circumstance is the practice's note recoverable.

Course qualification:

- For students receiving ongoing evaluation. The course note (NC), which is applicable if PR >= 4 and ((E1+E2) /2) >= 3.5, is defined as: NC = (4E1+4E2+2PR) /10.
- For the single evaluation students. The course grade (NC), which is applicable if PR >=4 and SE>=3.5, will be NC = (8SE+ 2PR) /10.
- In either of the two aforementioned scenarios, 80% of the grade can be made up on the final exam (FE). In this situation, the course grade will be NC = (8FE+ 2PR) /10 if PR>=4 and FE>=3.5.
- The possible grade with honors will be awarded based on the course syllabus. In the recovery of the assignment, no honorifics will be given.
- A student will be considered NO evaluable for the course if they have participated in evaluation activities that account for more than 50% of the grade according to the established weighting.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final test	80%	4	0.16	4, 1, 3, 2
First test	40%	3	0.12	4, 1, 3, 2
Practical exercices	20%	0	0	4, 1, 3, 2
Second test	40%	4	0.16	4, 1, 3, 2
Single examination (SE) - for the students who has requested a single evaluation method (4h)	80%	0	0	4, 1, 3, 2

Bibliography

For the course's first section, the required reading list will be as follows:

- "Ecuaciones diferenciales, sistemas dinámicos y àlgebra lineal", Morris W. Hirsch, Stephen Smale, Alianza Universidad Textos, Madrid, 1983.
- "Equaçoes Diferenciais Ordinarias", J. Sotomayor.
- "Qualitative Theory of Planar Differential Systems", Freddy Dmortier, Jaume Llibre, Joan C. Artés, Universitext, Springer, 2006.

Regarding the second and third topics:

- "Primer curso de ECUACIONES EN DERIVADAS PARCIALES", Ireneo Peral, UAM, Madrid, 1995. (pdf accessible a la web del professor)

- "EDP, um curso de graduaçao", Valéria Iório, IMPA, Brasil, 2001.
- "Partial Differential Equations Vol I", M.E. Taylor, Applied Mathematical Sciences, 2011.

As an auxiliary bibliography for the three topics, the following is suggested:

- "Models amb Equacions Diferencials", R. Martínez. Materials de la UAB no. 149. Bellaterra, 2004
- "Equaçoes Diferenciais: Teoria Qualitativa", L. Barreira i C. Valls, IST Press Lisboa 2010.
- "Ecuaciones Diferenciales y Cálculo Variacional ", Lev Elsgoltz, Mir, Moscou, 1983.
- "Apunts d'Equacions Diferencials", d'en Francesc Mañosas, UAB (available through the virtual campus)
- "Ecuaciones diferenciales", V. Jimenez. Serie: enseñanza. Universidad de Murcia, 2000.
- "Análise de Fourier e equaçoesdiferenciais parciais", Djaro guedes de Figueiredo, IMPA, Brasil, 2000.
- "Càlcul Infinitesimal amb Mètodes Numèrics iAplicacions", C. Perelló. Enciclopèdia Catalana, 1994.
- "Ecuaciones Diferenciales y Problemas con Valores en la Frontera", E. Boyce, y R.C. Di Prima, Ed. Limusa, México, 1967.
- "Partial Differential Equations, An Introduction", Walter Strauss, Wiley, New York, 1992.
- "Elliptic partial differential equations of second order", *David Gilbarg,* Berlin: springer, 1977.

Software

For the practical classes, we will use SAGE.