

Galois theory

Code: 100102
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	3	1

Contact

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Teaching groups languages

You can check it through this [link](#). To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Teachers

Pere Ara Bertran

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Prerequisites

Background on Group Theory is required (e.g. the notions introduced and studied in "Estructures Algebraiques"). Groups are used in an essential way in this module. Thus, in order to be able to work with concrete examples, it is particularly interesting to have some familiarity with groups of small order.

It is also important to be familiar with basic notions of Ring Theory (again, the ones introduced in "Estructures Algebraiques"). Of particular importance are the notions related to irreducible polynomials, as well as the construction of fields as quotients of polynomial rings.

Objectives and Contextualisation

The main objective of this module is to develop the notions of Galois Theory and their applications to problems related to resolubility of equations. The latter problems arise as some of the oldest in the History of Mathematics. Their roots can be traced back to the Babylonia era and culminates brilliantly with the work of Évarist Galois, whose work develops the theory of solvability by radicals.

The modern approach to Galois Theory constitutes a central theme in Algebra, since the abstract methods used show the power of (previously introduced) tools in action. Thus, the translation of a problem to Field Theory, and subsequently to Group Theory (and back) show how abstract, seemingly different branches of Mathematics interact to solve a classical, more applied problem.

We will start introducing the problem of solving an equation by radicals in its historical context. Next, Field Theory will provide the formal framework where to formulate the problem and study effectively the Galois Theory of equations.

A fundamental tool here is provided by the techniques coming from Group Theory, particularly when it comes to examples and manipulation. However, due to time constraints, we shall review only the most basic concepts and refer to the notions studied in the course "Estructures Algebriques".

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Calculate groups of low degree Galois equations and deduce their resolvability by radicals.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Manipulate expressions involving algebraic and transcendent elements.
5. Relate geometric constructions with algebraic extensions.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Basic Notions

Polynomial equations: the formulas for small degree.

Rings, ring homomorphisms. The field of fractions of a commutative domain.

Subrings and subfields generated by elements.

The characteristic of a field.

2. Field extensions

Algebraic and transcendental elements.

The degree of a field extension. The multiplicativity formula for degrees.

Algebraic extensions.

Extensions de homomorphisms. The group $\text{Gal}(L/K)$.

The splitting field of a polynomial.

Finite fields.

3. Normal and separable extensions

Normal extensions.

Formal derivative of a polynomial and polynomials of multiple roots.

Separable elements and separable extensions.

4. The Fundamental Theorem of finite Galois Theory

Galois extensions. Artin's Theorem.

Galois correspondence: the fundamental theorem.

5. Galois theory of equations.

Solvability by radicals and solvable groups.

Cyclotomic and cyclic extensions.

Galois Solvability Theorem .

Polynomials whose Galois group is S_p , where p is prime.

6. Fundamental theorem of algebra.

Methodology

There will be two lectures and one tutorial per week, during 15 weeks. In addition, there will be 3 seminar sessions of 2 hours each, distributed in the semester. Students are strongly encouraged to attend lectures, tutorials, and seminars.

During the lectures, the main tools needed for understanding the subject and also for problem-solving will be introduced.

Problem-solving will be the main focus in the tutorials, where also a better understanding of the concepts introduced in the lectures will be achieved. Students participation in the form of discussion will be part of the methodology.

In seminars, students participation will be more prominent as these are designed in the form of hands-on exercises and focusing, in particular, in manipulation of examples.

Various resources will be offered through moodle. In particular, problems/seminars and additional material that may complement the subject of the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	30	1.2	2, 3, 1, 4, 9, 8, 7, 6, 5
Seminars	6	0.24	2, 3, 1, 4, 9, 8, 7, 6, 5
Tutorials	15	0.6	2, 3, 1, 4, 9, 8, 7, 6, 5
Type: Autonomous			
Course work (from lectures)	27	1.08	2, 4, 9, 8, 6, 5
Exams preparation	16	0.64	2, 4, 8, 6, 5
Problem solving	40	1.6	2, 3, 4, 8, 6, 5
Seminar preparation	10	0.4	2, 3, 4, 8, 7, 6, 5

Assessment

The distribution of marks will be done as follows:

- 35% of the grade corresponds to the partial exam.
- 15% of the grade corresponds to seminars.
- 50% of the grade corresponds to the exam at the end of semester.

In case of unique evaluation, there will be an examen corresponding to 100% of the grade.

There will be a resit exam, that will substitute the grade corresponding to the partial exam and the final exam, in case that final grade is less than 5.

In case somebody does this resit exam, the final qualification will be 5 if the 85% of the grade obtained and 15% of the grade of the seminars is bigger or equal than 5, or the maximum of this qualification and the previous qualification in case it is less than 5.

In case of unique evaluation, the final qualification will be 5 if the grade obtained is bigger or equal than 5, or the maximum of this qualification and the previous qualification in case it is less than 5.

Qualification of Non-Assessed. A student will be classified as non-assessed if it does not assist to the final examen and to the resit exam.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Exam	50%	3	0.12	2, 3, 1, 4, 9, 8, 6, 5
Intersemester exam	35%	2	0.08	2, 3, 1, 4, 9, 8, 6, 5
Seminars	15%	1	0.04	2, 1, 4, 9, 8, 7, 6, 5

Bibliography

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<http://syndetics.com/index.aspx?isbn=0471434191/summary.html&client=autbaru&type=rn12>

D. S. Dummit, M. R. Foote, Abstract Algebra, Wiley, 2004.

D.J.H. Garling. A course in Galois Theory. Cambridge Univ. Press, 1986.

J. Milne. Fields and Galois Theory, <http://www.jmilne.org/math/>

P. Morandi. Fields and Galois Theory. GTM 167, Springer.

S. Roman. Field Theory. GTM 158, Springer.

Ian Stewart "Galois Theory" Chapman & Hall / CRC, 2004
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Additional bibliography:

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<http://syndetics.com/index.aspx?isbn=9780132413770/summary.html&client=autbaru&type=rn12>

T. Hungerford, "Algebra" New York : Springer-Verlag, cop. 1974
<http://syndetics.com/index.aspx?isbn=0387905189/summary.html&client=autbaru&type=rn12>

Jean-Pierre Tignol, "Galois' Theory of Algebraic Equations". World Scientific 2001

A. M. de Viola Priori, J.E. Viola-Priori. Teoría de cuerpos y Teoría de Galois. Reverté (2006).

Galois' life (novel, in Catalan):

Josep Pla i Carrera. Damunt les espatlles de gegants. 1ra Edició: Editorial la Magrana. 2na Edició: Edicions FME http://www.fme.upc.edu/ca/arxiu/damunt-les-espatlles-dels-gegants_jpla

Some interesting links:

<http://www.galois-group.net>
<http://godel.ph.utexas.edu/~tonyr/galois.html>
<http://www-groups.dcs.st-andrews.ac.uk/%7Ehistory/Mathematicians/Galois.html>
Curiosinats origami: Robert J. Lang: <http://www.langorigami.com>
Tom Hull: <http://www.merrimack.edu/thull/~omfiles/geoconst.html>
Koshiro Hatori: <http://origami.ousaan.com/library/conste.html>
<http://www.langorigami.com/science/mathlinks/mathlinks.php4>.

Software

We will use SageMath.