Universitat Autònoma de Barcelona

2023/2024

Dynamic Systems

Code: 100118
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	1

Contact

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Teaching groups languages

You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Prerequisites

Ordinary differential equations: existence and uniqueness of solutions of the Cauchy problem.

Linear differential systems with constant coefficients.

Linear algebra: spaces and vector subspecies, diagonalization.

Objectives and Contextualisation

This course is an introduction to the modern theory of dynamic systems. The first objective is to familiarize the student with the notion of a dynamical system and the basic concepts of this theory: stability, attractor, invariant sets, alpha and omega limits, etc. The second objective is to understand how is the local behavior, in discrete and continuous dynamical systems, near an equilibrium point or a periodic orbit. This local behavior is based on the topological classification of linear systems in R^n, both those that are determined by the flow of ordinary differential equations (continuous dynamical systems) and those that come from the iteration of functions (discrete dynamical systems). Linear systems are very important because they are the first approach to more complicated systems.

The Qualitative Theory of Differential Equations began with the work of Poincaré around 1880 in relation to his work on Celestial Mechanics and seeks to know properties of solutions without having to solve equations, among other things because the resolution is not feasible. This qualitative approach, when combined with appropriate numerical methods, is, in some cases, equivalent to having the solutions of the equation. Progress will be made in the knowledge and study, introduced previously on the plane, of the qualitative theory of differential equations in spaces of higher dimension. Emphasizing the local structure of equilibrium points (degenerate and non-degenerate) and the stability of their periodic orbits.

Finally, we introduce the techniques to study discrete global dynamics. The main example will be the unimodal maps. They (for some parameter values) present a dynamic that simply leads to the notion of a chaotic system. For these systems, the numerical approach is not feasible, and to understand its dynamics new tools are needed. Chaotic systems are often presented in applications (problems of weather forecasting, electrical circuits, etc.).

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Know how to apply the dynamical tools described in theory lectures to describe processes governed by differential equations.
4. Know how to demonstrate the results of partial derivative equations and dynamical systems.
5. Know how to solve certain theoretical problems and be understand the existence of certain open problems in the theory of partial derivative equations and dynamical systems theory.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Dynamical systems in Euclidean spaces.

- Dynamical systems defined by differential equations and by difeomorphisms.
- Orbits; Critical points and periodic orbits.
- Invariant sets and limit sets.

- Attractors. Liapunov stability.
- Conjugation of dynamic systems.

2. Study of local dynamics, discrete and continuous in R^n.

- Phase portraits near equilibrim and regular points.
- Topological classification of continuous and discrete linear systems.
- Stability (Liapunov's Functions)
- Hartman theorems, of the stable variety and of the central variety.
- Periodic orbits: Application of Poincaré and stability.

3. Global dynamics in continuous systems.

- Ordinary differential equations in R2 (Theorem of Poincaré-Bendixon, Theorem of Bendixon-Dulac, Existence and unicity of limit cycles, ...)
- Ordinary differential equations in dimension greater than 2.

4. Global dynamics in discrete systems.

- Iteration in dimensions 1 and 2.
- Unimodal applications.
- Chaos Bernoulli's shift. Smale's Horseshoe.

Methodology

The subject has, during the semester and per week, two hours of theoretical lessons and one hour more to help to solve the typical problems.

The schedule and classrooms can be consulted on the website of the degree course or in the Virtual Campus (CV) of the university. In it you will find some of the material and all the information related to this subject.

Theoretical lessons. The teacher will be developing the different parts of the program. The CV will also have available to the students a bibliography and support material, if necessary, for the theory and/or problems.

Solving problem lessons. The lists of problems to be solved will be available in the CV.

During the seminars, some concepts will be developed by the students.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem solving classes	14	0.56	
Seminars	6	0.24	
Theoretical lessons	29	1.16	
Type: Autonomous			

Exam Preparation	15	0.6
Problem solving	42	1.68
Study of the theoretical part	32	1.28

Assessment

See the Catalan version.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	45%	3	0.12	2, 5, 1, 9, 8, 7, 6, 3, 4
Partial exam	35%	3	0.12	5, 9, 3, 4
Second-chance Examination	45%	0	0	9
Seminaris (3 activities)	20%	6	0.24	2, 1, 9, 8, 7, 3

Bibliography

See the Catalan version.

Software

The student will be able to use any of the programming languages that he has knowledge (C, Sagemath, Maxima, Maple, Mathematica, ...). Knowledge of some symbolic computing software will be useful.