Universitat Autònoma de Barcelona

2023/2024

Vector Spaces

Code: 104343
ECTS Credits: 6

Degree	Type	Year	Semester
2503758 Data Engineering	FB	1	2

Contact

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Teaching groups languages

You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Prerequisites

No required.

Objectives and Contextualisation

In order to acquire a good mathematical training in the treatment of data, it is essential to fully understand the theory of Vector Spaces. It is necessary to learn to manipulate the objects that are introduced and interpret its meaning. The tools that are provided in this course are essential not only in all branches of Mathematics but also in most of the engineering.

Among the training objectives we emphasize the following: understand and correctly use mathematical language, see the need for demonstrations and develop a critical sense in the face of mathematical affirmations.

As more specific objectives: the student will learn to manipulate arrays as a basic tool to analyze systems of linear equations, formalize the language necessary to understand the concepts of vector space and linear application, as well as manipulate bilinear forms. Diagonalization in linear applications, and some application in the world of data engineering. All this might be reinforced by the introduction of certain software.

Competences

- Demonstrate sensitivity towards ethical, social and environmental topics.
- Make a critical evaluation of work carried out.
- Search, select and manage information and knowledge responsibly.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Use the concepts and methods of algebra, differential and integral calculus, numerical methods, statistics and optimisation necessary for solving engineering problems.

Learning Outcomes

1. Be able to manipulate matrices.
2. Calculate and interpret the meaning of the representations given by projection in a vector subspace.
3. Demonstrate sensitivity towards ethical, social and environmental topics.
4. Make a critical evaluation of work carried out.
5. Search, select and manage information and knowledge responsibly.
6. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
7. Understand the concept of vector space, basis and linear representation both in finite dimension spaces and in infinite dimension spaces.

Content

The subject is structured in four blocks: a first more computational block where the algebraic manipulation of matrices is prioritized, introducing their basic operations. In the second block the concepts of abstract vector space and linear application will be formalized, relating them to the contents of the first block. The third block presents a factorization in linear applications that has different uses in the world of engineering. The fourth block is dedicated to more advanced concepts that take advantage of the structure of vector space with metrics.

Topic 1: Matrices and linear equations

(A) Operations with matrices. Invertible matrix.

(B) Elemental transformations in matrices.

(C) Rank of a matrix. Invertibility criterion. PAQ-reduction. Generalized Invers matrix.

(D) Resolution of systems of linear equations.

(E) Determinant of a square matrix.

Topic 2: Vector spaces and linear applications

(A) Definition of space and vector subspace. Scalar products in vector spaces. Linear independence, generators and bases. Dimension.

(B) Nucleus and image of a linear application. Composition.

(C) Vector coordinates and matrix associated with a linear application.

Topic 3: Diagonalization

(A) Characteristic polynomial. Eigenvalues.

(B) Eigenvectors associated with an eigenvector. Diagonalization of matrices.

(C) Minimum polynomial.

Topic 4: Orthogonality, normed spaces and quadratic forms.

(A) Bilinear forms and diagonalization in symmetric matrices.

(B) Singular values and SVD factoring (Singular Value Decomposition). Fitting Date.

(C) Hilbert spaces.

Methodology

The subject has during the semester of 4 weekly hours grouped in blocks of 2 hours. Each of these blocks will be divided into a theoretical introduction of content and problem solving, which may be on paper or with the use of software.

To introduce the software, more time will be devoted to this part of the sessions at the beginning of the course.

During lectures or tutorials, in the last half hour of the 2-hour block, and without notice, there will be (4 times in different days) a small test that students must do individually, which will count towards evaluation.

This course will have the corresponding Moodle classroom within the UAB servers to offer complementary material.

Professors should allocate approximately 15 minutes of some class to allow their students to answer the surveys for the evaluation of teaching performance and the evaluation of the subject or module.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Solving exercises and Computer class with a mathematic programme.	24	0.96	4, 5, 2, 1, 7
theory class	26	1.04	2, 1, 3, 7
Type: Autonomous			
Computer work with Sage Math	22	0.88	5, 1, 7
Learn theoretical concepts and solving exercises.	61	2.44	5, 2, 1, 7
Preparing exercises to be handed in	10	0.4	5, 2, 1, 7

Assessment

Continuous assessment:

During the course there will be 1 individual hand-in of a list of exercises that will be posted in Moodle a week before. Students must submit the resolution of the list individually. The note of this hand-in can not be recovered, we call this a grade A (out of 10 points).

During the course, and without previous notice, half an hour of the regular lecture or the tutorials will be devoted to a small quiz, based on the contents of each block. It will be done individually, in the classroom. There will be 4 Quizzes, one per block. The grades of these tests are not recoverable either. Each Quiz will have an equal score, and the average between 0 and 10 will be denoted by B.

Exam type evaluation:

During the month of December, at a time and date to be fixed, there will be a computer-based practice assessment. The level achieved with the course will be assessed with the help of software with the laptop. The test will be individual. This test can be retaken during the resit date, however it must have a minimum score of 1 point out of 10 to be able to evaluate the course; otherwise the student will fail the course (see the grading section). We denote this grade between 0 and 10 by P, and remember it is mandatory to take this test since P must be greater than or equal to 1 in order to pass the course.

At the end of the course, there will be a final exam of the whole subject. Denote by E the final exam grade on 10 points.

Qualification of the subject (without resits):

If the grade E is equal to or higher than 3.5 and the grade P is equal to or higher than 1, then at this point the student has the qualification N = 0.1 * A + 0.25 * B + 0.15 * P + 0.5 * E. If the grade is higher than or equal to 5, the student passes the course with note N.

If P <1 or E <3,5 (or the student hasn't turned up at the practical exam or end of the subject) the student obtains the minimum grade between N and 4.5 points.

The student obtains a Non-evaluable in case she or he does fails to hand in exercises, does not turn up for the last two Quizzes and she or he does not turn up in any of the exams.

Single assessment: Students who decide to take the single assessment will take a written test on the same day as the final exam in which the contents of the entire course will be assessed, including those contents covered in the exercises delivered. Those students will also take another test, on the same day, of practicals with a computer. These tests can be recovered on the day of the recovery exam, in the same format.

Resit exams:

Students with N<5 or E<3.5 (always with P>1) must take the make-up exam if they want to try to pass the subject, otherwise the grade will remain as described above. To be able to take the recovery exam it is imperative that P>1.

The make-up exam is an exam of the whole course with the same value as the final exam of the course, we say in the note of this make-up exam by Erec.

Final qualification of the course (for students that take the resit exam):

We denote by Nfin:=0.1*A +0.25*B+0.15*P+0.5*Erec. If Erecup>3.5, the student's qualification will be Nfin. If Erecup is lower than 3.5, the student's grade will be the minimum between 4.5 and Nfin.

Annex on the qualification of the subject:

Students who have more than a 9.25 in the final qualification will have a Matricula d'Honor (MH) until reaching the limit of 5% of those enrolled. If there are more than 5% of students above 9.25, those with the highest marks will have MH.

"Without prejudice to other disciplinary measures that are deemed appropriate, andin accordance with current academic regulations, irregularities committed by a student that may leadto a variation of the qualification will be graded with a zero (0). The activities of 'assessment graded in this way and by this procedure will not be recoverable. If it is necessary to pass any of these assessment activities to pass the subject, this subject will be suspended directly, with no opportunity to recover it in the same course. These irregularities include, among others:
- the total or partial copy of a practice, report, or any other assessment activity;
- let copy;
- present a group work not done entirely by the group members;
- present as own materials prepared by a third party, even if they are translations or adaptations, and in general works with non-original and exclusive elements of the student;
- have communication devices (such as mobile phones, smart watches, etc.) accessible during individual theoretical-practical assessment tests (exams)."

In case of discrepancy, the version that maintains validity is the version in Catalan.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	50%	3	0.12	2, 1, 7, 6
Hand-in of a list of exercises	10%	0	0	4, 5, 2, 1, 7, 6
Quiz	25%	2	0.08	4, 5, 2, 1, 3, 7, 6
SageMath exam	15%	2	0.08	4, 1, 7, 6

Bibliography

Bretscher,O. "Linear Algebra with Applications", 1997, Prentice-Hall International, Inc.

Nart,E.;Xarles,X."Apunts d'àlgebra lineal", 2016, col.lecció Materials UAB, num.237.

Seasone,G"Elementary notions of Hilbert Spaces" 1991, NewYork, Dover.

Virtual Bibliography:

Bars, F.:Uns apunts de càlcul matricial i resolució de sistemes lineals. https://ddd.uab.cat/record/73660

Bars.F: Una pinzellada del polinomi mínim. https://ddd.uab.cat/record/236746

Bars, F: Espais normats i Espais de Hilbert, per a primer curs. https://ddd.uab.cat/record/236744

Software

Use of SageMath with the computations inputs of the different subjects given in the course.