Universitat Autònoma de Barcelona

2023/2024

Linear Algebra

Code: 104381
ECTS Credits: 6

Degree	Type	Year	Semester
2503740 Computational Mathematics and Data Analytics	FB	1	1

Contact

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Teaching groups languages

You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Teachers

Albert Ruiz Cirera

Francesc Bars Cortina

External teachers

Pol Orobitg

Prerequisites

Although the course is essentially self-contained, the student will be required to know how to solve systems of linear equations, the basic arithmetic of numbers and polynomials, and how to perform algebraic manipulations.

Objectives and Contextualisation

In order to acquire a proper mathematical training, it is essential to understand linear algebra in depth. One needs to learn how to manipulate the objects introduced in such a class and to interpret their meanings. The tools provided in this course are essential not only in all branches of Mathematics, but also in most Sciences and Engineering studies.

Among the many goals we underline the following: to understand and correctly use mathematical language, to appreciate the need for proofs, and to develop a critical approach to mathematical statements.

As more specific goals: the student will learn to manipulate matrices as a basic tool to analyze systems of linear equations, to formalize the necessary language in order to understand the concepts of vector space and

linear map, as well as to manipulate bilinear forms. All of this will be reinforced with the introduction of the appropriate software.

Competences

- Apply a critical spirit and rigour for the validation or rejection of your own arguments and those of others.
- Calculate and reproduce certain mathematical routines and processes with ease.
- Demonstrate a high capacity for abstraction and translation of phenomena and behaviors to mathematical formulations.
- Formulate hypotheses and think up strategies to confirm or refute them.
- Make effective use of bibliographical resources and electronic resources to obtain information.
- Relate new mathematical objects with other known objects and deduce their properties.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Using criteria of quality, critically evaluate the work carried out.
- Work cooperatively in a multidisciplinary context assuming and respecting the role of the different members of the team.

Learning Outcomes

1. "Explain ideas and mathematical concepts pertinent to the course; additionally, communicate personal reasonings to third parties."
2. Apply a critical spirit and rigour for the validation or rejection of your own arguments and those of others.
3. Calculate determinants and decompositions for matrices.
4. Calculate orthonormal bases and projections.
5. Contrast, if possible, the use of calculation with the use of abstraction in solving a problem.
6. Describe the concepts and mathematical objects pertaining to the subject.
7. Develop autonomous strategies for solving problems such as identifying the ambit of problems within the course, discriminate routine from non-routine problems, design an a priori strategy to solve a problem, evaluate this strategy.
8. Evaluate the advantages and disadvantages of using calculation and abstraction.
9. Identify the essential ideas in the demonstration of ceratin basic theorems and know how to adapt these to obtain other results.
10. In an orderly and accurately manner, draft brief mathematical texts (exercises, resolution of theoretical questions, etc.).
11. Make effective use of bibliographical resources and electronic resources to obtain information.
12. Read and understand a mathematical text at the current level of the course.
13. Solve and discuss systems of linear equations.
14. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
15. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
16. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
17. Using criteria of quality, critically evaluate the work carried out.
18. Work cooperatively in a multidisciplinary context, taking on and respecting the role of the distinct members in the team.

19. Work with different bases of finite-dimension vector spaces.

Content

The course is structured in 4 parts: a first, more computational one, where the emphasis is put in the algebraic manipulation of matrices, introducing their basic operations. In the second part the concepts of abstract vector space and linear map are formalized, as well as their relation with the contents of the first block. The third and fourth parts are devoted to more advanced concepts that build on the structure of vector space and linear map.

Parts

1. Matrices and linear equations
2. Vector spaces and linear maps
3. Diagonalization
4. Orthogonality and quadratic forms

Methodology

The course has 4 weekly hours grouped in blocks of 2 hours during the semester. Each of these blocks will combie theoretical contents and resolution of problems, which may be on paper or with the use of software.

In order to introduce the software, more time will be devoted to this part in the first sessions.

During the course five quizzes will be offered, and students will have to take them individually. The dates in which these tests will be announced at the beginning of the course.

This course will realize, initially, in-class lectures and also taking advantage of the resources made available by UAB. It will also make extended use of the corresponding Moodle classroom hosted at UAB's servers, to complement the explanations made in class, offer the necessary material, open forums and make deliveries.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	27.5	1.1	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11
Practice sessions	11	0.44	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11
Problem sessions	12	0.48	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11
Type: Autonomous			
Preparation of problems to deliver	15	0.6	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11
Problem solving	30	1.2	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11

Theory study	26	1.04	2, 17, 8, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 19, 18, 11
Use of software	20	0.8	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11

Assessment

During the course, the dates of each test or evaluation will be announced. There will be second oportunity for the final exam and the practice exam. The student needs a grade bigger to 3,5 for the final exam in order to compute grade by using the marks of the regular quizes, problems to turn in, and practice exam.

The mark corresponding to *Regular quizzes* will be obtained from the average of the three best qualifications out of the five tests that will be made, and will not be recoverable.

There will be two problems that the student must do on their own and turn in.

Students who have requested the single assessment and it has been accepted by the Faculty, the assessment will consist of a final exam that will count for 80% of the grade and a practical exam that will count for 20% and that both they have a minimum grade of 3,5 points to be able to pass the subject. These exams will be taken one after the other on the date of the subject's final exam.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	50%	4	0.16	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 11
Practice exam	15%	2	0.08	17, 8, 4, 3, 5, 6, 7, 1, 9, 16, 15, 14, 13, 19, 11
Problems to turn in	15%	0	0	2, 17, 8, 4, 3, 5, 6, 7, 1, 9, 12, 16, 15, 14, 10, 13, 19, 18, 11
Regular quizzes	20%	2.5	0.1	2, 17, 8, 4, 3, 5, 6, 7, 16, 15, 14, 13, 19, 18, 11

Bibliography

Teaching notes:

- Marc Masdeu, Albert Ruiz, *Apunts d'Àlgebra Lineal*. Available at the Moodle classroom.

Basic:

- Otto Bretscher, *Linear Algebra with Applications*. Pearson, 2013.
- Enric Nart, Xavier Xarles, *Apunts d'àlgebra lineal*. Materials UAB, 2016.

Complementary:

- Sheldon Axler, *Linear algebra done right*. Springer UTM, 2015.
- Manuel Castellet i Irene Llerena, *Àlgebra lineal i geometria*. Manuals UAB, 1991.
- Ferran Cedó i Agustí Reventós, *Geometria plana i àlgebra lineal*. Manuals UAB, 2004.
- - Gilbert Strang, Linear Algebra and Learning from Data. Wellesley-Cambridge Press, 2019, pp.446. ISBN:978-06921963-8-0
  - Mike X. Cohen, Practical Linear Algebra for Data Science: From Core Concepts to Applications using Python. O'reilly Media, 300pp (2022). ISBN:978-1098120610
  - Charu C. Aggarwal, Linear algebra and optimization for Machine Learning: a textbook. Springer International Publishing (2020). ISBN: 9783030403430

Software

SageMath (https://www.sagemath.org/)