2023/2024

Equations in Partial Derivatives

Code: 104401
ECTS Credits: 6

Degree	Type	Year	Semester
2503740 Computational Mathematics and Data Analytics	OB	3	1

Contact

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Teaching groups languages

You can check it through this link. To consult the language you will need to enter the CODE of the subject. Please note that this information is provisional until 30 November 2023.

Teachers

Carles Barril Basil

Prerequisites

It is convenient that the student has already passed the subjects *Ordinary differential equacions* and *Calculus in more than one variable*

Objectives and Contextualisation

Partial differential equations are a fundamental tool in deterministic modeling of problems in physics, engineering, biology, medicine or finance, among others. The objective of the course is a first introduction to these equations both from an analytical and numerical point of view. We will start with the first order equations by first studying the most basic aspects of the characteristics method for quasi-linear equations. Some of the applications of these models, such as the traffic equation, will be used to visualize the difficulties of modeling and the appearance of weak solutions. Later, the "typical" second order linear equations of mathematical physics will be studied: wave, heat and Laplace. As with ordinary differential equations, in very few cases there are closed formulas available for solving partial differential equations, which is why numerical methods are required to approximate the solutions. In this course, the finite difference method will be introduced as a numerical approximation of the solutions of some of the equations studied.

Competences

- Calculate and reproduce certain mathematical routines and processes with ease.

- Design, develop and evaluate efficient algorithmic solutions to computational problems in accordance with the established requirements.
- Formulate hypotheses and think up strategies to confirm or refute them.
- Relate new mathematical objects with other known objects and deduce their properties.

Learning Outcomes

1. Contrast, if possible, the use of calculation with the use of abstraction in solving a problem.
2. Describe the concepts and mathematical objects pertaining to the subject.
3. Develop demonstrations of the mathematical results from numerical calculation and the numerical integration of EDP.
4. Evaluate and analyse the complexity of computational algorithmic solutions in order to develop and implement that which guarantees best performance.
5. Numerically integrate ordinary differential equations and partial differential equations.
6. Programme mathematical-calculation algorithms.
7. Verify and ensure the correct operation of an algorithmic solution in accordance with the requirements of the problem to be resolved.

Content

1. Introduction and first definitions.

2. First order partial differential equations.
Linear and quasilinear PDEs with two variables. The transport equation. Characteristics Method. Application to structured population dynamics.
Conservation laws. Bürgers equation and traffic equation. Rarefaction waves, weak solutions and shocks. Entropy conditions.
Finite difference method for hyperbolic equations.

Topic 3. The wave equation.
Vibrating string equation. D'Alembert's formula. Domain of dependence and domain of influence.
The vibrating membrane. Linear waves in electromagnetism. Explicit formulas of the solution in dimensions 2 and 3.

Topic 4. The heat equation.
The heat equation. Linear diffusion. Existence of solution for Cauchy's problem: Poisson's formula.
The maximum principle: uniqueness of solution.
Finite differences for the heat equation.

Item 5. The potential equation.
Harmonic functions. The Dirichlet and Neumann problems. Green functions.

Methodology

This course consists of two hours of theory class per week. In addition, 10 hours of seminar will be held where students will solve exercises proposed by the teacher. There will be 12 hours of practical classes that will be devoted mainly to the approximate calculation of solutions of partial differential equations. All the material and all the necessary information for the development of the subject will be provided in the Virtual Campus.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Theory lessons	27	1.08	1, 2, 3
Type: Supervised			
Practical classes	12	0.48	6, 7
Seminars	10	0.4	4, 3, 5
Type: Autonomous			
Problem solving and practices	40	1.6	4, 1, 2, 3, 5, 6, 7
Study	55	2.2	4, 1, 2, 3, 5, 7

Assessment

The following evaluation activities will be carried out:

Partial exam (EP). Exam with theoretical questions and problems similar to those worked on during the course.
Final Exam (EF). Exam of the whole subject with theoretical questions and problems similar to those worked on during the course.
Practice mark (PR). It will be evaluated from the project (program) and the corresponding report.

In addition, students will be able to take a recovery exam (ER) with the same characteristics as the exam (EF). The practice mark will not be recoverable.
It is a requirement to pass the subject that max (0.35 * EP + 0.65 * EF, EF, ER)> = 3.5 and that PR> = 3.5.

The final grade for the course will be
0.65 * max (0.35 * EP + 0.65 * EF, EF, ER) + 0.35 PR
The honors will be awarded in the first evaluation in which the subject can be passed.

A student who has participated in assessment activities corresponding to less than 50% of the grade according to the established weight will be considered non-evaluable.

Students who have taken the single assessment modality must take the subject's final exam (EF) on the same date as students taking the continuous assessment. This test will account for 65% of the grade. On this same date, the student will have to evaluate the project and report (PR) and, if the teacher requires it, an oral evaluation of the project and report will take place. This evaluation will account for 35% of the final grade. If the final grade is lower than 5, the student can take the recovery exam (ER) with the same characteristics as the exam (EF). The practice grade will not be recoverable. It is a requirement to pass the subject that max(EF, ER)>=3.5 and that PR>=3.5.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	40%	3	0.12	4, 1, 2, 3, 5, 7
Partial Exam	25%	3	0.12	4, 1, 2, 3, 5, 7
Practice Delivery	35%	0	0	4, 1, 2, 3, 5, 6, 7

Bibliography

- Y. Pinchover and J. Rubinstein. An introduction to partial differential equations. 2005.

- I. Peral, Primer Curso de EDPs, Addison-Wesley/UAM, 1995.

- L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics 19, AMS, 1998.

- S. Salsa, *Partial Differential Equations in action: from modelling to theory,* Springer, 2008.

- F. John, Partial Differential Equations, Springer-Verlag, 1980.

- W. A. Strauss, Partial Differential Equations: An Introduction, John Wiley \& Sons, 1992.

- J. C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, SIAM 2004.

- R. Haberman. Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow. 1998.

Software

The practice delivery will be done in R but it might be allowed to use other programming languages.