UAB
Universitat Autònoma de Barcelona

Galois theory

Code: 100102
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OB	3

Contact

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Teachers

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Teaching groups languages

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Prerequisites

Background on Group Theory is required (e.g. the notions introduced and studied in "Estructures Algebraiques"). Groups are used in an essential way in this module. Thus, in order to be able to work with concrete examples, it is particularly interesting to have some familiarity with groups of small order.

It is also important to be familiar with basic notions of Ring Theory (again, the ones introduced in "Estructures Algebraiques). Of particular importance are the notions related to irreducible polynomials, as well as the construction of fields as quotients of polynomial rings.

Objectives and Contextualisation

The main objective of this module is to develop the notions of Galois Theory and their applications to problems related to resolubility of equations. The latter problems arise as some of the oldest in the History of Mathemathics. Their roots can be traced back to the Babylonia era and culminates brilliantly with the work of Évarist Galois, whose work develops the theory of solvability by radicals.

The modern approach to Galois Theory constitutes a central theme in Algebra, since the abstract methods used show the power of (previously introducted) tools in action. Thus, the translation of a problem to Field Theory, and subsequently to Group Theory (and back) show how abstract, seemingly different branches of of Mathematics interact to solve a classical, more applied problem.

We will start introducing the problem of solving an equation by radicals in its historical context. Next, Field Theory will provide the formal framework where to formulate the problem and study effectively the Galois Theory of equations.

A fundamental tool here is provided by the techniques coming from Group Theory, particularly when it comes to examples and manipulation. However, due to time constraints, we shall review only the most basic concepts and refer to the notions studied in the course "Estructures Algebraiques".

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Calculate groups of low degree Galois equations and deduce their resolvability by radicals.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Manipulate expressions involving algebraic and transcendent elements.
5. Relate geometric constructions with algebraic extensions.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Solvability of equactions and ring plreliminaires

2. Field extensions

3. Normal and separable extensions

4. The Fundamental Theorem of finite Galois Theory

5. Galois theory of equations.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Lectures	30	1.2	2, 3, 1, 4, 9, 8, 7, 6, 5
Seminars	6	0.24	2, 3, 1, 4, 9, 8, 7, 6, 5
Tutorials	15	0.6	2, 3, 1, 4, 9, 8, 7, 6, 5
Type: Autonomous			
Course work (from lectures)	27	1.08	2, 4, 9, 8, 6, 5
Exams preparation	16	0.64	2, 4, 8, 6, 5
Problem solving	40	1.6	2, 3, 4, 8, 6, 5
Seminar preparation	10	0.4	2, 3, 4, 8, 7, 6, 5

There will be two lectures and one tutorial per week, during 15 weeks. In addition, there will be 3 seminar sessions of 2 hours each, distributed in the semester. Students are strongly encouraged to attend lectures, tutorials, and seminars.

During the lectures, the main tools needed for understanding the subject and also for problem-solving will be introduced.

Problem-solving will be the main focus in the tutorials, where also a better understanding of the concepts introduced in the lectures will be achieved. Students participation in the form of discussion will be part of the methodology.

In seminars, students participation will be more prominent as these are designed in the form of hands-on exercises and focusing, in particular, in manipulation of examples.

Various resources will be offered through moodle. In particular, problems/seminars and additional material that may complement the subject of the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Exam	50%	3	0.12	2, 3, 1, 4, 9, 8, 6, 5

Intersemester exam	35%	2	0.08	2, 3, 1, 4, 9, 8, 6, 5
Seminars	15%	1	0.04	2, 1, 4, 9, 8, 7, 6, 5

The subject will be evaluated as follows:

- 35% of the grade will correspond to the completion of a partial exam.
- 15% of the grade will correspond to seminar evaluation.
- 50% of the grade will correspond to the completion of a final exam.

In the case of a single assessment, there will be a final exam corresponding to 100% of the final grade that will be held to coincide with the date of the final exam.

There will be a second chance exam, both for the continuous assessment and for the single assessment, which will make it possible to recover the grade of the exams in the event that the average of the subject is lower than 5.

The grade of non-evaluable will be obtained only if neither the final exam nor the retake is taken.

Bibliography

F.Bars, Teoria de Galois en 30 hores, http://mat.uab.cat/~francesc/docencia2.html

David A. Cox, Galois Theory. Hoboken : Wiley-Interscience, cop. 2004
http://syndetics.com/index.aspx?isbn=0471434191/summary.html&client=autbaru&type=rn12

Jean-Perre Tignol, "Galois' Theory of Algebraic Equations". World Scientific 2001

D.J.H. Garling. A course in Galois Theory. Cambridge Univ. Press, 1986.

J. Milne. Fields and Galois Theory, http://www.jmilne.org/math/

P. Morandi. Fields and Galois Theory. GTM 167, Springer.

S. Roman. Field Theory. GTM 158, Springer.

Ian Steward "Galois Theory" Chapman & Hall / CRC, 2004
http://syndetics.com/index.aspx?isbn=1584883936/summary.html&client=autbaru&type=rn12

Additional bibliography:

Michael Artin, "Algebra" Prentice Hall, cop. 2011
http://syndetics.com/index.aspx?isbn=9780132413770/summary.html&client=autbaru&type=rn12

T. Hungerford, "Algebra" New York : Springer-Verlag, cop. 1974
http://syndetics.com/index.aspx?isbn=0387905189/summary.html&client=autbaru&type=rn12

A. M. de Viola Priori, J.E. Viola-Priori. Teoría de cuerpos y Teoría de Galois. Reverté (2006).

Software

We will use SageMath.

Language list

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	first semester	morning-mixed
(PAUL) Classroom practices	2	Catalan	first semester	morning-mixed
(SEM) Seminars	1	Catalan	first semester	morning-mixed
(SEM) Seminars	2	Catalan	first semester	morning-mixed
(TE) Theory	1	Catalan	first semester	morning-mixed