UAB Universitat Autònoma de Barcelona

Complex and Fourier analysis

Code: 100103
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OB	3

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

It is a third-year subject; therefore, the students already have a certain mathematical background necessary to follow it. Although it will be quite self-contained, certain prior knowledge is essential. For example, the theory of series and power series and the improper integrals of Mathematical Analysis and the differential calculus in several variables. Although some aspects of complex numbers have already been seen in other courses, they will be repeated here to facilitate students' learning.

Objectives and Contextualisation

Know and be able to use the fundamental concepts and results of Complex Analysis.

Understand in depth the demonstrations of the most important results and the most common techniques in the area.

Have some initial notions of the basic concepts of the Fourier transform and the Laplace transform.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Recognise the presence of Mathematics in other disciplines.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Comfortably deal with the calculation of residues and its applications
4. Contrast acquired theoretical and practical knowledge.
5. Handle with ease homographic transformations and conformal representation.
6. Know how to calculate Fourier periodic function coefficients and their possible immediate applications for calculating sums of series.
7. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
9. Understand Fourier and Laplace transforms of elemental functions and their application to the resolution of differential equations.
10. Understand the basic results and fundamental properties of holomorphic functions and Cauchy theory.

Content

1. Preliminaries. Complex numbers. Power series. Holomorphic functions. Cauchy-Riemann equations.

2. Cauchy's Local Theory. Complex line integrals. Cauchy-Goursat theorem and the local Cauchy theorem. Holomorphy and analyticity. Zeros of holomorphic functions. The index of a closed curve. Cauchy's integral formula. Analytical extension. Cauchy inequalities, Liouville's theorem and Fundamental theorem of algebra. The principle of the maximum module. Schwarz's motto.

3. Singularities. Laurent series. Classification of isolated singularities. Waste theorem and applications. The principle of argument and Rouché's theorem.

4. Harmonic functions. Basic properties of harmonic functions. Harmonic functions on a disc. Dirichlet problem.

5. Integral transforms. Fourier transform. Laplace transform. Basic properties. Applications to solve equations.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercices	14	0.56	2, 4, 10, 9, 1, 3, 5, 6
Maths seminar	6	0.24	2, 4, 10, 9, 1, 3, 5, 8, 7, 6
Theory	28	1.12	2, 10, 9, 1, 3, 5, 8, 7, 6
Type: Autonomous			

Studying time	88	3.52	2, 4, 10, 9, 1, 3, 5, 6

The subject has two hours of theory per week. They will be taught in the traditional way with chalk and blackboard. In the theory where the concepts will be broken down and the important results (theorems) that underpin the theory we are introducing will be stated.

We will demonstrate theorems and solving methods through examples and exercises.

The student will receive lists of exercises and problems that we will work on in the weekly problem class. Previously, during your off-site activity, you will have read and thought about the proposed exercises and problems. In this way, their participation in the classroom can be guaranteed and the assimilation of procedural content will be facilitated.

There will be three seminar sessions, each lasting two hours. The students will have material previously placed on the Virtual Campus that they must have studied. In the first two sessions there will be a (short) first part where the teacher will add some details about the content of the practice. Then the students will get to work on a list of activities. Practices can be done in pairs, which seems to help them a lot. The third session of the seminars will be assessable. The planned topics are a more in-depth study of Möbius transformations and more applications of the residue theorem in the calculation of definite integrals. The evaluation will deal with these topics.

The Virtual Campus will be the means of communication between teachers and students. It will be important to consult it every day.

The students will have a tutoring and counseling service in the office. It is recommended to use this aid for monitoring the course.

Note: 15 minutes of a class will be reserved within the calendar established by center/degree for completion by students in the evaluation surveys of the teaching staff's performance and evaluation of the subject/module .

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	40	4	0.16	2, 4, 10, 1, 3, 8, 7
Maths Seminars	20	2	0.08	10, 9, 3, 5, 6
Recovery exam	80	4	0.16	2, 4, 10, 9, 1, 3, 5, 8, 7, 6
Second partial exam	40	4	0.16	10, 9, 3, 8, 7, 6

Learning mathematics is a complex process. A maturation is needed that is achieved throughout the course. Many times, some result of the beginning of the theory comes to be completely understood very advanced in the course. This shows the difficulty of assessments.

There will be two partial written exams during the semester, which will mainly consist of solving problems, but will also contain a theoretical part. They will be rated P1 and P2 respectively. The exam qualification will be their mean E.

The seminar test will assign an S grade of up to 20%.

The grade for continuous assessment will be obtained with the formula

QC= 0.8*E+0.2*S.

The teacher may interview the students in order to modify the latest qualification.

During the course the teacher may offer the possilibity to obtain extra points for certain tasks. For instance if the students writes say a report and obtains a qualification T, then the final qualification would be QC= 0.7*E+0.1*max(E,T)+0.2*S

If QC is greater than or equal to 5 the course will be passed. Otherwise, the student will be able to present himself for a make-up, and will obtain an R and a grade

QC'= min(0.8* R+0.2*S,5),

i.e.,only a 5 can be achieved in the make-up test.

The grade will always be

QF=maximum{QC, QC'}.

The possible honors registrations will be awarded respecting the regulations in force and once the entire evaluation has been completed, possible recovery included.

If a student has only taken an assessment test, they will be given "Not assessable" as the final grade.

AVALUACiÓ ÚNICA:

People who, for very justified reasons, cannot take the continuous assessment, can take the single assessment. This option must be requested with the requirements set by the degree. Accepted people will be able to take the exam together with the make-up exam in which two questions about the seminars will be added, they willget a grade S. Their grade will be

QU=0.8*R+0.2*S

If QU exceeds 3.5 and does not reach 5, the student will have a recovery option under the same conditions, but only 5 can be achieved. If QU does not exceed 3.5, this will be the grade that will be awarded.

Bibliography

Bibliografia bàsica:

1) L. Ahlfors, Complex Analysis. Mc Graw-Hill. 3ra edició, 1979.(És una referència clàssica que amb un format reduït tracta moltíssims temes de forma rigorosa).

2) J. Conway, Functions of One Complex Variable, second Edition, Springer Verlag, 1978. (Abarca molt més que el curs i conté molts problemes).

3) J. P. D'Angelo; An introduction to Complex Analysis and Geometry; A.M.S. 2010 (És una introducció de nivell molt més elemental que els anteriors).

4) B. Davis; Transforms and Their Applications, Thrid Edition, Springer (2001) (Serveix com a inici i aprofundiment en l'estudi del món de les transformacions integrals).

5) M. C. Pereyra and L. A. Ward. *Harmonic Analysis: From Fourier to Wavelets*, AMS, 2012 (Curs força complet d'anàlisi harmònica)

Bibliografia complementària:

1) J. Bruna, J. Cufí, Anàlisi Complexa, Manuals UAB 49, 2008.

2) R. Burckel, Introduction to classical complex Analysis, vol I, Academic Pres, 1979.

3) W. Rudin, Análisi Real y Complexo, Alhambra, 1979

4) S. Saks et A. Zygumund, Fonctions Analytiques, Massin et Cie, 1970.

5) M. Stein, R: Shakarchi, Complex Analysis, Princeton University Press, 2003.

Software

There are no computer practice classes in the subject, so no study of computer programs will be done. Despite this, it will be recommended to use mathematical manipulation programs such as Sagemath, Maxima or Wolfram Alpha, which can be very useful.

Language list

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(PAUL) Classroom practices	2	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	morning-mixed
(SEM) Seminars	2	Catalan	second semester	morning-mixed
(TE) Theory	1	Catalan	second semester	morning-mixed