UAB
Universitat Autònoma de Barcelona

Real and functional analysis

Code: 100110
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OT	4

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

All the previous courses of Calculus and Mathematical Analysis.

Good knowledge of Linear Algebra and Basic Topology is also important.

Objectives and Contextualisation

Explain the concepts and fundamental results of the Lebesgue integral in Euclidean space.

Present the methods of functional analysis, in the context of Banach spaces and Hilbert spaces.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
3. Confidently deal with the most important Hilbert spaces and know how to apply the basic theory of Functional Analysis to them.
4. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
5. Devise demonstrations of mathematical results in the field of mathematical analysis.
6. Effectively use bibliographies and electronic resources to obtain information.
7. Formulate conjectures and devise strategies to confirm or reject said conjectures
8. Generate innovative and competitive proposals for research and professional activities.
9. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
10. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
11. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
12. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
13. Understand the concept of R^n measurement and its construction process.
14. Understand the language and in-depth demonstrations of some advanced mathematical analysis theorems.
15. Understand the nature of the Lebesgue integral and its advantages over the Riemann integral.

Content

The course consists of 3 blocks:

Theory of Measure, Banach Spaces and Hilbert Spaces.

1. Limitations of the Riemann integral.

2. Lebesgue measure. Abstract measure theory.

3. Lebesgue integral. Abstract integral theory. Limit vs. integral.

4. Fundamental Theorem of Calculus. Variable change theorem. Fubini-Tonelli theorem.

5. Integrals dependent on a parameter. Differentiating under the integral sign.

6. Normed spaces. Banach spaces. Characteristics.

7. Spaces of sequences. Spaces of functions. Spaces of measures.

8. Bounded linear operators. Norm of an operator. Topology of bounded linear operators.

9. Applications: Volterra's integral equation.

10. Open Mapping Theorem and Closed Graph Theorem. Uniform boundedness principle.

11. Dual topological of a normed space. Hahn-Banach theorem.

12. Hilbert spaces. Theorem of the Projection. Orthogonality

13. Hilbertian basis. Bessel inequality. Parseval's identity.

14. Fourier series. Riemann-Lebesgue lemma.

15. Compact operators. Sturm-Liouville problem.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercices lessons	14	0.56	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Theorical lessons	30	1.2	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Type: Supervised			
Seminars	6	0.24	2, 14, 15, 4, 13, 7, 5, 3, 12, 10, 6
Type: Autonomous			
Personal study	92	3.68	

This subject has 2 hours of theory and 1 of problems per week.

It also consists of a total of 6 hours of seminars throughout the course.

Although it is not compulsory, it is highly recommended to attend classes to ask questions and venture answers, even if they are incorrect.

Theory: we will develop the main results and put them in the context of future applications.

Problems: students will receive some lists of exercises that we will solve in problem classes.

Seminars: will serve to complement the contents of theory and problems.

Students will also have a few hours of consultation in the teacher's office, to consult questions, discuss methods, etc.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Block 1. Measure Theory	25%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 10, 6
Block 2. Banach Spaces.	25%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6
Block 3. Hilbert Spaces	25%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6
Delivery of exercises	25%	2	0.08	2, 14, 15, 1, 4, 13, 7, 8, 5, 3, 12, 11, 9, 10, 6

During the course we will do an evaluation activity (two hours) for each block. It will consist in presenting the demonstration of some result, of a list established before the evaluation, and in the resolution of exercises.

Block 1. Measure Theory (25%)

Block 2. Banach Spaces (25%)

Block 3. Hilbert Spaces (25%)

The delivery of solved exercises, as the teacher did, indicating, complements (25%) the course evaluation.

On the day designated by the Coordination of the Degree as a Final Exam (or recovery), students who have not passed the course will take a make-up exam with all the course material. The maximum score that can be obtained in this recovery exam is 7.

ALL THE CONTENTS OF THE COURSE ARE EVALUABLE (THEORY, PROBLEMS, SEMINARS).

For each assessment activity, a place, date and time of review will be indicated in which students can review the activity with the teaching staff. In this context about the grade of the activity, which will be evaluated by the teaching staff responsible for the subject. Students who do not attend this review will not be able to review the activity at a later date.

One-time evaluation:

Students who have opted for the single assessment mode must take a final exam consisting of a theory exam where they will have to develop a topic and/or answer a series of short questions. They will then have to take a problem/practical test where they will have to solve a series of exercises similar to those that have been worked on in the Practical/Classroom Problem sessions.
The grade will be the weighted average of the two previous activities, where the theory exam will account for 30% of the grade and the problem/practical exam for 70%.
If the final grade does not reach 5, the failed students have another opportunity to pass the subject by means of the make-up exam that will be held on the date set by the degree coordination. In order to take the make-up exam, students must have obtained a minimum grade of 3.5.
The review of the final grade follows the same procedure as for continuous assessment.
The proposed teaching methodology and assessment may undergo some modification depending on the restrictions imposed by the health authorities.
This English version of the guide is a translation of the Catalan version. In the event of any discrepancy between the two, the correct version for all purposes is the Catalan version.

Bibliography

J. Bruna, *Anàlisi Real*, UAB Servei de Publicacions, 1996.

J.M. Burgués, *Integració i càlcul vectorial,* UAB Servei de Publicacions, segona edició, 2002.

J. L. Cerdà Martín, Análisis Real, Col·lecció UB 23, segona edició, 2000.

J. L. Cerdà Martín, Introducció a l'Anàlisi Funcional, Textos Docents 280, Publicacions i edicions UB, 2005.

W. Rudin, F*unctional analysis,* Alambra,1979.

Software

We will not use any specialized software.

Language list

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	first semester	morning-mixed
(SEM) Seminars	1	Catalan	first semester	afternoon
(TE) Theory	1	Catalan	first semester	morning-mixed