Riemannian geometry

Code: 100115
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OT	4

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Teachers

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Teaching groups languages

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Prerequisites

For good follow-up of the subject it is recommended a good assimilation of the concepts introduced in the subject Geometría Diferencial.
Analysis knowledge (Calculation in various variables and optimization), topology (Topology) and differential equations (Differential Equations and Modeling I) will also be used.

Objectives and Contextualisation

A Riemannian manifold is a differentiable manifold endowed with a scalar product in the tangent space at each point. Riemannian geometry is the study of Riemannian manifolds and first appears as a generalization of the intrinsic geometry of surfaces. Later it turns out to be the perfect paradigm where classical mecanic and general relativity can be formulated. More recently this topic was decisive in the proof of Poincaré's conjecture albeit the topological nature of this conjecture.

The two main notions in Riemannian geometry are curvature and geodesics. The main objective of this course is to understand, geometrically and as far as possible, the interconnexion between these two notions. In particular we will analyze the effect of curvature on geodesics and on the topology of manifolds.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Demonstrate a high capacity for abstraction.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
4. Devise demonstrations of mathematical results in the field of geometry and topology.
5. Effectively use bibliographies and electronic resources to obtain information.
6. Generate innovative and competitive proposals for research and professional activities.
7. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
8. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
9. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
10. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
11. Understand abstract language and in-depth demonstrations of some advanced theorems of geometry and topology.

Content

1. Riemannian manifolds. Notion of Riemannian length and volume.
2. Connections. Geodesics. Exponential map and Gauss Lemma. The Hopf-Rinow theorem.
3. Curvature. Jacobi fields.
4. Hyperbolic geometry.
5. Other topics in Riemannian geometry.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Theoretical course	30	1.2	2, 11, 9
Tutorials	14	0.56	2, 11, 4, 9
Type: Supervised			
Seminars	6	0.24	2, 11, 4, 9, 8
Type: Autonomous			
Personal study	45	1.8	2, 11, 9, 5
Preparation and exhibition of works	16	0.64	2, 11, 9, 8, 5
Resolution of problems	30	1.2	2, 4, 9, 8, 5

The subject has two weekly hours of theory class and one of problems. In addition, throughout the course there will be three seminars of two hours each.

- In the *theory* classes, the fundamental notions of Riemannian geometry will be introduced and the most important results of the theory will be presented. Likewise, the necessary tools for the understanding and resolution of problems will be given.
- The *problems* classes will be deepened in the assimilation and the understanding of the concepts developed in the theoretical classes will be improved by solving theoretical problems and exercises designed to increase the students' dexterity in their own calculations. This work will be carried out through explanations made by the teacher on the board and the active participation of students in the discussion of the different arguments used to solve the problems. The lists of problems will be delivered to the students throughout the semester.
- The *seminars* will be devoted to deepening issues dealt with in theory classes and problems. Students will receive a script before each seminar will be held. During the session, they will have to work autonomously, although they can be consulted by the teachers. Subsequently, they will deliver the solution to the problems worked during the seminar. These solutions will be corrected by the teachers, giving rise to a part of the continuous evaluation note.

At the same time, each student will draw up a work on a chosen subject from a list proposed by the teachers. This work will be delivered in writing, as well as exposing yourself in class. The evaluation of both aspects (written and oral) will also be part of the continuous evaluation.

Individual tutorials, or in small groups, are foreseen for students who want it in the professor's office.
In the end the student will have received the theory and problems classes, as well as the seminars, all the necessaryinformation (both the statements and their demonstrations), to face the partial test such as the final test. This subject will also offer resources through the Virtual Campus.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Delivery of problems	0,2	2	0.08	2, 11, 1, 4, 10, 9, 8, 5
Exam	0,3	2.5	0.1	2, 11, 1, 4, 9, 8, 5
Presentation of works	0,2	2	0.08	2, 11, 3, 6, 10, 9, 8, 7, 5
Recovery test	0,3	2.5	0.1	2, 11, 1, 4, 9, 8, 5

The evaluation of this subject will take into account the assimilation of the contents, as well as the work done during the course, and will be carried out in the form of a continuous evaluation.

The final grade will be obtained by weighted average between the score obtained in the partial exam module (40%), the problem delivery module (30%) and the presentation module of work (30%).

The possible license plates will be awarded based on the continuous evaluation note.

Students who had not passed the continuous evaluation, that is to say that they had not obtained a final grade equal or superior to five, or that they want to improve their mark, will have a final test of recovery.
A student will be qualified as "Not Presented" if the weight of the assessment activities in which he has participated does not exceed 50% of the weight of the continuous assessment of the subject.

Unique assessment

Students who have accepted the single assessment modality will have to take a final test which will consist of an exam. These tests will be carried out on the same day, time and place as the examination of the continuous assessment modality. When finished, you will hand in the required assignments and deliverables in the continuous assessment activities.

The student's grade will be the weighted average of the previous activities, where the exam will account for 40% of the grade and the assignments and assignments 60%.

If the final grade does not reach 5, the student has another opportunity to pass the subject through the remedial exam that will be held on the date set by the degree coordinator. The part of work and deliveries is not recoverable.

Bibliography

1- Manfredo P. do Carmo, Riemannian Geometry. Birkhäuser, 1992.
2- Manfredo P. do Carmo, Geometŕia diferencial de curvas y superficies. Alianza Universidad, 1990.
3- S. Gallot, D. Hulin, J. Lafontaine, Riemannian Geometry. Springer-Verlag, 1990.
4- Joan Girbau, Geometria diferencial i relativitat. Manuals de la UAB, Servei de Publicacions de la U.A.B.,1993.
5- John M. Lee, Riemannian Manifolds: An introduction to curvature. Springer-Verlag, 1997.
6- M. Spivak, A Comprehensive Introduction to Diferential Geometry. Publish or Perish Inc, 1979.
7- J. Cheeger, D. Ebin, Comparison Theorems in Riemannian Geometry. Noth Holland, 1975.

Software

We will not use software in this course.

Language list

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	afternoon
(TE) Theory	1	Catalan	second semester	morning-mixed