UAB
Universitat Autònoma de Barcelona

Stochastics processes

Code: 100116
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OT	4

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

As a general requirement, in order to be able to follow this subject, good knowledge is required at the practical level of linear algebra, analysis and calculus or, more specifically, matrices, integration and series. As more specific requirements, it is necessary to have previously studied a course in Probability and Stochastic Modeling.

Objectives and Contextualisation

The aim of this subject is, on the one hand, to introduce the student in the part of the theory of probability called theory of stochastic processes, whose purpose is to study the random phenomena that evolve over time or in the space We will see the basic generalities of these models and we will study some specific models.

The discrete Markov chains will be studied in general and in the particular case of the random walk. We will also study the continuous-time Markov chains, such as the Poisson process or the birth and death processes. Finally we will also introduce the Brownian motion.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Effectively use bibliographies and electronic resources to obtain information.
- Generate innovative and competitive proposals for research and professional activities.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Devise demonstrations of mathematical results in the field of probability and statistics.
3. Effectively use bibliographies and electronic resources to obtain information.
4. Generate innovative and competitive proposals for research and professional activities.
5. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
6. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
7. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

0. Introduction and preliminaries.

1. Discrete-time Markov chains.
1.1. Definitions. Basic properties Transition matrix.
1.2. Stoping time Strong Markov property.
1.3. Recurrent and transient states. Equivalence classes. More aspects of the random walk.
1.4. Asymptotic behavior. Invariant distribution. Ergodic theorem.

2. Markov chains in continuous time.
2.1. Motivation: The Poisson process.
2.2. Basic properties Generating Matrix. Kolmogorov differential equations
2.3. Class structure and classification of states.
2.4 Invariant distribution. Ergodic theorem.

3. Brownian motion.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem classes	13	0.52	1, 4, 2, 7, 6, 3
Theory classes	28	1.12	1, 2, 7, 3
Type: Supervised			
Seminars	6	0.24	1, 4, 2, 7, 6
Type: Autonomous			
Prepare exams	20	0.8	2, 7, 3
Study of the theory and resolution of problems	65	2.6	1, 4, 2, 7, 6, 3

This subject is semiannual and consists of two hours of theory and one hour of problems per week. There will also be three two-hour seminars.

In theory classes, the teacher plays the main role and we will work with the structure definition-theorem-proof-application.

In the problem classes, practical problems will be solved.

On the other hand, in the seminar sessions, the student will work, under the tutelage of the teacher, some practical situations that are related to what has been studied in the theory classes.

These sessions will also allow, students and teachers, to be aware of the evolution in the achievement of the concepts and methods that are introduced in theory classes.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	(1-0.1·x)·5%(<50%)	4	0.16	1, 2, 7, 6, 5
Quizz	10·x%	6	0.24	1, 4, 2, 7, 6, 3
Recovery exam	100%	4	0.16	1, 4, 2, 7, 6, 3
Second partial exam	(1-0.1·x)·5%(<50%)	4	0.16	1, 2, 7, 6

During the semester, two partial exams will be held. The first will take place approximately halfway through the course and the second will take place at the end of the course. The final grade of the exams, y, will be obtained by averaging the two partials' grades.
Throughout the course they will be performed some quizz in class hours with which an x note will be obtained. Additional tasks can also be proposed during the course that will contribute to the x grade. The final note of the subject will be obtained by applying the following formula:
$N(x,y)=x+(1-0.1\cdot x)\cdot y$
We refer to the article *Matemáticas y evaluación*, Materials Matemàtics volum 2011, by X. Bardina and E. Liz in which the formula is explained in detail.
If necessary, a recovery test will be scheduled.

Single Assessment

Students who have taken part in the single assessment procedure will have to carry out a final test, which will consist of a test-type theory test. Next they will have to do two problem tests corresponding to the two partials of the subject.

The student's mark will be the weighted average of the three previous activities, where the theory exam will be 20% of the mark, and the problem exams 40% each.

Bibliography

1. Bardina, X. & Ferrante, M. An excursion into Markov chains. Springer, *to appear*.
2. Breiman, L. Probability and Stochastic Processes: With a ViewToward Applications. Houghton Mifflin Company Boston, 1969.
3. Brémaud, P. Markov Chains: Gibbs measures, Montecarlo simulation, and queues. Texts in Applied Mathematics. Springer, 1998.
4. Feller, W. Introducción a la teoría de probabilidades y sus aplicaciones, Vol I. John Wiley \& Sons, 1988.
5. Karlin, S. & Taylor, M.H. A First Course in Stochastic Processes. Academic Press, New York, 1975.
6. Karlin, S. & Taylor, M.H. A Second Course in Stochastic Processes. Academic Press, New York, 1981.
7. Lawler, G.F. Introduction to Stochastic Processes. Chapman and Hall/CRC Probability Series, 1995.
8. Norris, J.R. Markov Chains. Cambridge University Press, 1997.
9. Hoel, P.G., Port, S.C. & Stone, C.J. Introduction to Stochastic Processes. Houghton Mifflin Company, Boston, 1972.

Software

No specific software is needed for this course.

Language list

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	afternoon
(TE) Theory	1	Catalan	second semester	morning-mixed