

## Partial differential equations

Code: 100119 ECTS Credits: 6

2024/2025

Degree	Туре	Year
2500149 Mathematics	ОТ	4

#### Contact

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**Teachers** 

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## **Teaching groups languages**

You can view this information at the <u>end</u> of this document.

## **Prerequisites**

(See the catalan official version)

## **Objectives and Contextualisation**

(See the catalan official version)

### Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Students must be capable of applying their knowledge to their work or vocation in a professional way
  and they should have building arguments and problem resolution skills within their area of study.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

• Understand and use mathematical language.

## **Learning Outcomes**

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- 2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- 3. Know how to demonstrate the results of partial derivative equations and dynamical systems.
- 4. Know how to solve certain theoretical problems and be understand the existence of certain open problems in the theory of partial derivative equations and dynamical systems theory.
- 5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- 6. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- 7. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

#### Content

- 1. Introduction to partial differential equations
- 1.1. Partial differential equations in Science, Technology and Finance.
- 1.2. Basic concepts: order, linearity.
- 1.3. Mathematical Physics equations: the heat equation, the wave equation, the potential equation. Initial conditions and boundary conditions. Stationary problems.
- 2. First-order partial differential equations
- 2.1. Linear and quasi-linear first-order partial differential equations with two variables. The method of characteristic curves. The initial value problem.
- 2.2. Introduction to conservation laws. The traffic equation. Some initial value problems. Rarefaction waves and shocks. Entropy condition.
- 2.3. First order nonlinear equations.
- 3. Second-order semi-linear partial differential equations
- 3.1. Canonical forms of second-order semi-linear equations with two variables. Classification.
- 4. Cauchy's problem. Sofia Kovalevskaia's theorem
- 5. The wave equation
- 5.1. One-dimensional waves. d'Alembert's formula. Zones of influence and dependence.
- 5.2. The wave equation in 2 and 3 dimensions.
- 6. The heat equation
- 6.1. The equation of heat in the straight line and in space. Poisson's formula. regularity.
- 6.2. The beginning of the maximum principle. Uniqueness of solution.
- 7. The potential equation
- 7.1. The harmonic functions, properties
- 7.2. The Dirichlet and Neumann problems. Uniqueness of solution
- 7.3. Green's functions
- 7.4. The Dirichlet principle and the variational method. Generalized solutions.

### **Activities and Methodology**

Title	Hours	ECTS	Learning Outcomes
Type: Directed			

Lectures	30	1.2
Type: Supervised		
Problem sessions and working seminars	21	0.84
Type: Autonomous		
Problem solving	34	1.36
Studying theoretical concepts	50	2

This subject consists of 2 weekly hours of theory class, one of problems and three seminars of two hours each.

In the theory classes, the various types of equations in partial derivatives, their derivation, the characteristic properties of each type of equation, the methods of resolution and the various concepts of solution will be exposed.

The problem classes will be dedicated to solving practical problems that will be developed on the board. For this, work will be done on lists of problems that will be provided to the student in advance throughout the semester and that will also be uploaded to the website.

There will also be three seminars of two hours each, where specific topics will be deepened and a problem will be solved in a guided way.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

### Assessment

### **Continous Assessment Activities**

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	40%	4.5	0.18	2, 4, 1, 7, 6, 5, 3
Second partial exam	40%	4.5	0.18	2, 4, 1, 7, 6, 5, 3
Seminars	20%	6	0.24	2, 4, 1, 7, 6, 5, 3

In principle, the subject will be assessed through two partial exams, which will count for 40% each, and the evaluation of the seminars, which will count for 20%. However, it will be necessary to meet the condition that the grades obtained in the partial exams are both higher than or equal to 3. If this condition is not met, or the grade obtained is lower than 5, then it will be possible to opt for a recovery exam that will replace the two partial exams and will count for 80%. The evaluation of the seminars is not recoverable.

Possible honor degrees will be awarded based on the overall marks resulting from the two partial exams and the seminars, that is, without waiting for the recovery exam. If this does not exhaust the number of possible honor degrees, then the remaining ones may be awarded after the recovery examination.

In the event that the single assessment is chosen, this will be carried out on the day of the second partial exam but will have a value of 100%. The exam will contain an exercise related to the seminars. The student will have the right to a new exam if he/she has not passed the previous one, on the day of the recovery exam, under the same conditions.

# **Bibliography**

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### **Software**

No specific soiftware will be used

# Language list

		1		
Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	first semester	morning-mixed
(SEM) Seminars	1	Catalan	first semester	morning-mixed
(TE) Theory	1	Catalan	first semester	morning-mixed