UAB
Universitat Autònoma de Barcelona

Numerical integration of partial differential equations

Code: 100121
ECTS Credits: 6

2024/2025

Degree	Type	Year
2500149 Mathematics	OT	4

PROVISIONAL

Contact

Name: Angel Calsina Ballesta

Email: angel.calsina@uab.cat

Teachers

Miquel Barcelona Poza

Teaching groups languages

You can view this information at the end of this document.

Prerequisites

This course has no theoretical prerequisites, although courses on partial differential equations and numerical analysis would help to provide context. The practical work requires a minimum familiarity with the use of the C programming language for scientific computing.

Objectives and Contextualisation

This course is an introduction to numerical methods for the solution of partial differential equations (PDE).

PDE are in the basis of most mathematical models of physical processes. As with ordinary differential equations (ODE), closed-formulae solutions are available in very few cases. Because of that, in almost all applications numerical methods are required for the approximation of their solutions. Contrary to ODE, though, there are no general numerical methods applicable to almost all PDE except for some special behaviours: the methods are specific for small families of PDE. The ideas giving rise to the methods are general, and, in this way, we can speak of families of methods, like finite difference methods or finite element methods.

The course will be focussed on the development and analysis of finite difference and finite element methods for the classical PDE (transport, waves, heat and potential), although some comments will be made on other methods (such as characteristics or spectral) and other equations.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
3. Devise demonstrations of mathematical results of numeric calculus and numeric integration of PDEs.
4. Generate innovative and competitive proposals for research and professional activities.
5. Know how to numerically integrate ordinary differential equations and partial derivative equations.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1.- Finite differences

Hyperbolic evolution problems. The transport equation. Consistency, stability and convergence. Local truncation error and order of a method. The Courant-Friedrichs-Lewy condition.

Parabolic evolution problems. Explicit and implicit methods. The method of John Crank i Phyllis Nicolson. Stability

Stationary problems. The Poisson equation.

2.- Finite elements

Variational formulation. Stages: meshing, assembly, solution of the linear system, post-processing. Example: the 2D Poisson equation.

PROVISIONAL

Triangulations. Interpolation in several variables, families of finite elements. Boundary conditions. Assembly and global formulation.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercise classes	10	0.4	1, 2, 3, 8, 6, 5
Practical classes	14	0.56	1, 2, 3, 8, 6, 5
Theory classes	26	1.04	1, 2, 3, 8, 5
Type: Autonomous			
Problems solving and practices	44	1.76	1, 2, 3, 8, 6, 5
Study	50	2	1, 2, 3, 8, 5

The sessions on theory and problems will be carried out in a classroom. These sessions will consist in the presentation of the methods and their properties and the solutio of problems of a theoretical nature. Problem lists with be provided during the course.

The practical sessions will be carried out in a computer room. In these sessions, students will solve an applied problem through the implementation in a programming language of some of the methods studied in the course. These sessions will be evaluated from the delivery at the end of the course (the exact date will be announced) of the code and a report.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Delivery of problems	0.05	0	0	1, 2, 3, 9, 8, 6, 5
Final exam	0.45	3	0.12	2, 3, 9, 8, 6, 5
Practice delivery	0.5	0	0	1, 2, 4, 9, 8, 7, 6, 5
Recovery exam	0.5	3	0.12	2, 8, 6, 5

The evaluation of the course will be carried out from three activities:

- Final exam (FE): it's an exam of the whole course with theoretical questions and problems.
- Practical work (PR): delivery of code and a report.

- Optional delivery of problems: code and a report.

Additionally, students will be able to take an additional recovery exam RE which will be analogous to the FE exam. Practical work will not be recoverable.

In order to succeed, it is required that max(FE, RE)>=3.5 and PR>=3.5

The weighing of exams and practices will be

0.5*max(FE, RE)+0.5*PR

Students will be given the option to deliver some problems of the problems list designed to experiment in a computer with the properties of some of the methods studied in the course. The evaluation of these problems may add one point (out of 10) to the EF and ER grades. In any case, the maximum score will be 10.

Honor grades will be awarded at the first complete evaluation. They will not be revoked in the case that another student obtains a higher grade after consideration of the RE exam.
A student that takes part in an evaluation activity different from just exercises delivery will be cosidered assessable.
The student who opts for the single assessment will have an exam on the day of the final exam in which he/she will have to deliver the report and practice code. In case he/she does not pass the exam, ther will be a recovery exam on the same day as the classmates, under the same conditions as described above.

Bibliography

- C. Johnson: Numerical Solution of Partial Differential Equations by the Finite Element Method. Dover, 2009.

- M. G. Larson, F. Benzgon: The finite element method: Theory, implementation and applications. Springer, 2013.

- J. Masdemont: Curs d'elements finits i aplicacions. Edicions UPC, 2002.

- R.M.M. Mattheij, S.W. Rienstra, J.H.M. ten Thije Boonkkamp: Partial Differential Equations. Modeling, Analysis, Computation. SIAM, 2005.

- K.W. Morton, D.F. Mayers: Numerical Solution of Partial Differential Equations, Cambridge University Press, 1994.

- J. C. Strikwerda: Finite difference schemes and partial differential equations, SIAM, 2004.

Software

- Preferably a Linux environment
- code-oriented text editor (e.g. Kate)
- GNU C compiler
- gnuplot
- image manipulation tools (e.g. imagemagick)
- GNU Octave

Language list

PROVISIONAL

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(PLAB) Practical laboratories	1	Catalan	second semester	morning-mixed
(TE) Theory	1	Catalan	second semester	morning-mixed

PROVISIONAL