

## Equations in Partial Derivatives

Code: 104401  
ECTS Credits: 6

2024/2025

Degree	Type	Year
2503740 Computational Mathematics and Data Analytics	OB	3

### Contact

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### Teachers

Carles Barril Basil

### Teaching groups languages

You can view this information at the [end](#) of this document.

### Prerequisites

It is convenient that the student has already passed the subjects *Ordinary differential equations* and *Calculus in more than one variable*

### Objectives and Contextualisation

Partial differential equations are a fundamental tool in deterministic modeling of problems in physics, engineering, biology, medicine or finance, among others. The objective of the course is a first introduction to these equations both from an analytical and numerical point of view. We will start with the first order equations by first studying the most basic aspects of the characteristics method for quasi-linear equations. Some of the applications of these models, such as the traffic equation, will be used to visualize the difficulties of modeling and the appearance of weak solutions. Later, the "typical" second order linear equations of mathematical physics will be studied: wave, heat and Laplace. As with ordinary differential equations, in very few cases there are closed formulas available for solving partial differential equations, which is why numerical methods are required to approximate the solutions. In this course, the finite difference method will be introduced as a numerical approximation of the solutions of some of the equations studied.

### Learning Outcomes

1. KM10 (Knowledge) Describe the mathematical concepts and objects of differential equations and numerical methods.

2. KM10 (Knowledge) Describe the mathematical concepts and objects of differential equations and numerical methods.
3. KM11 (Knowledge) Devise demonstrations of mathematical results of numerical calculus and numerical integration of ordinary differential equations and partial differential equations.
4. KM11 (Knowledge) Devise demonstrations of mathematical results of numerical calculus and numerical integration of ordinary differential equations and partial differential equations.
5. SM11 (Skill) Numerically integrate ordinary differential equations and partial differential equations.

## Content

1. Introduction and first definitions.

2. First order partial differential equations.

Linear and quasilinear PDEs with two variables. The transport equation. Characteristics Method. Application to structured population dynamics.

Conservation laws. Burgers equation and traffic equation. Rarefaction waves, weak solutions and shocks.

Entropy conditions.

Finite difference method for hyperbolic equations.

Topic 3. The wave equation.

Vibrating string equation. D'Alembert's formula. Domain of dependence and domain of influence.

The vibrating membrane. Linear waves in electromagnetism. Explicit formulas of the solution in dimensions 2 and 3.

Topic 4. The heat equation.

The heat equation. Linear diffusion. Existence of solution for Cauchy's problem: Poisson's formula.

The maximum principle: uniqueness of solution.

Finite differences for the heat equation.

Item 5. The potential equation.

Harmonic functions. The Dirichlet and Neumann problems. Green functions.

## Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Theory lessons	27	1.08	KM10, KM11, KM10
Type: Supervised			
Practical classes	12	0.48	KM10, KM11, SM11, KM10
Seminars	10	0.4	KM10, KM11, SM11, KM10
Type: Autonomous			
Problem solving and practices	40	1.6	KM10, KM11, SM11, KM10
Study	55	2.2	KM10, KM11, SM11, KM10

This course consists of two hours of theory class per week. In addition, 10 hours of seminar will be held where students will solve exercises proposed by the teacher. There will be 12 hours of practical classes that will be devoted mainly to the approximate calculation of solutions of partial differential equations. All the material and all the necessary information for the development of the subject will be provided in the Virtual Campus.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Assessment

### Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	40%	3	0.12	KM10, KM11
Partial Exam	25%	3	0.12	KM10, KM11
Practice Delivery	35%	0	0	KM10, KM11, SM11

The following evaluation activities will be carried out:

Partial exam (EP). Exam with theoretical questions and problems similar to those worked on during the course.

Final Exam (EF). Exam of the whole subject with theoretical questions and problems similar to those worked on during the course.

Practice mark (PR). It will be evaluated from the project (program) and the corresponding report.

In addition, students will be able to take a recovery exam (ER) with the same characteristics as the exam (EF). The practice mark will not be recoverable.

It is a requirement to pass the subject that  $\max(0.35 * EP + 0.65 * EF, EF, ER) \geq 3.5$  and that  $PR \geq 3.5$ . In case the student does not meet these conditions, the final grade will be 3.5.

The final grade for the course will be

$$0.65 * \max(0.35 * EP + 0.65 * EF, EF, ER) + 0.35 PR$$

The honors will be awarded in the first evaluation in which the subject can be passed.

A student who has participated in assessment activities corresponding to less than 50% of the grade according to the established weight will be considered non-evaluable.

Students who have taken the single assessment modality must take the subject's final exam (EF) on the same date as students taking the continuous assessment. This test will account for 65% of the grade. On this same date, the student will have to evaluate the project and report (PR) and, if the teacher requires it, an oral evaluation of the project and report will take place. This evaluation will account for 35% of the final grade. If the final grade is lower than 5, the student can take the recovery exam (ER) with the same characteristics as the exam (EF). The practice grade will not be recoverable. It is a requirement to pass the subject that  $\max(EF, ER) \geq 3.5$  and that  $PR \geq 3.5$ .

## Bibliography

- Y. Pinchover and J. Rubinstein. An introduction to partial differential equations. 2005.
- I. Peral, Primer Curso de EDPs, Addison-Wesley/UAM, 1995.
- L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics 19, AMS, 1998.
- S. Salsa, *Partial Differential Equations in action: from modelling to theory*, Springer, 2008.
- F. John, Partial Differential Equations, Springer-Verlag, 1980.
- W. A. Strauss, Partial Differential Equations: An Introduction, John Wiley & Sons, 1992.
- J. C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, SIAM 2004.
- R. Haberman. Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow. 1998.

## Software

The practice delivery will be done in R but it might be allowed to use other programming languages.

## Language list

Name	Group	Language	Semester	Turn
(PLAB) Practical laboratories	1	Catalan	first semester	morning-mixed
(SEM) Seminars	1	Catalan	first semester	morning-mixed
(TE) Theory	1	Catalan	first semester	morning-mixed