UAB
Universitat Autònoma de Barcelona

Partial Different Equations: Modelling, Analysis and Numerical Approximation

Code: 44211
ECTS Credits: 6

2024/2025

Degree	Type	Year
4313136 Modelling for Science and Engineering	OT	0

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Teachers

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

Students should have basic knowledge of calculus, algebra and ordinary differential equations, as well as basic notions of programming.

Objectives and Contextualisation

Many phenomena that unfold in space and/or time can be modelled by means of partial differential equations. The purpose of this course is to provide the main concepts about such models as well as numerical methods for computing their solution.

Competences

- Analyse, synthesise, organise and plan projects in the field of study.
- Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
- Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
- Apply techniques for solving mathematical models and their real implementation problems.
- Formulate, analyse and validate mathematical models of practical problems in different fields.
- Isolate the main difficulty in a complex problem from other, less important issues.
- Present study results in English.
- Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
- Use appropriate numerical methods to solve specific problems.

Learning Outcomes

1. Analyse, synthesise, organise and plan projects in the field of study.
2. Apply logical/mathematical thinking: the analytic process that involves moving from general principles to particular cases, and the synthetic process that derives a general rule from different examples.
3. Apply partial derivative equation techniques to predict the behaviour of certain phenomena.
4. Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialisation.
5. Extract information from partial derivative models in order to interpret reality.
6. Identify real phenomena as models of partial derivative equations.
7. Isolate the main difficulty in a complex problem from other, less important issues.
8. Present study results in English.
9. Solve complex problems by applying the knowledge acquired to areas that are different to the original ones.
10. Solve real problems by identifying them appropriately from the perspective of partial derivative equations.
11. Use appropriate numerical methods to study phenomena modelled with partial derivative equations.

Content

PART I: PDE MODELS AND THEIR MAIN PROPERTIES

I.0. Introduction: Examples, different types of equations.

I.1. The heat equation. The solution formula for the pure initial value problem; the Gauss kernel. Solution by means of the Fourier method in the case of a bounded interval with Dirichlet or Neumann boundary conditions. Dissipative character of the heat equation. The parabolic maximum principle.

I.2. The wave equation. The solution formula for the pure initial value problem. Solution by means of the Fourier method in the case of a bounded interval with Dirichlet or Neumann boundary conditions. Conservative character of the wave equation.

I.3. Laplace's equation with Dirichlet or Neumann boundary conditions. Variational principle. The elliptic maximum principle. The Poisson kernel. Solution by means of the Fourier method in the case of a rectangle, a circle or a sphere.

I.4. Turing's "chemical basis of morphogenesis".

I.5. Travelling-wave solutions of non-linear heat equations.

I.6. The traffic equation and scalar conservation laws. Shocks. Weak solutions. Rankine-Hugoniot and entropy conditions.

I.7. The Navier-Stokes equations.

PART II: NUMERICAL METHODS

II.1. Finite difference methods for scalar parabolic equations: Euler explicit, Euler implicit and Crank-Nicholson methods: Von Neumann stability test. Parabolic stability Courant-Friedrichs-Lewy condition. Examples.

II.2. Numerical methods for elliptic equations.

II.3. Numerical methods for scalar conservation laws: Finite difference methods in conservation form. Shock-capturing schemes. Monotone schemes: Lax-Friedrichs and upwind schemes. Convergence and stability conditions. Entropy-satisfying schemes. Examples.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Classes of theory and problems	30	1.2	5, 6, 10
Type: Supervised			
Internship classes	8	0.32	11
Type: Autonomous			
Studies and practical work by the student.	96	3.84	5, 6, 10

The aim of the classes of theory, problems and practices is to give to the students the most basic knowledge of tl

and their applications.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	30%	4	0.16	2, 1, 4, 3, 8, 7, 5, 6, 9, 10, 11
Second partial exam	30%	4	0.16	10
Solution of a problem with a computer	40%	8	0.32	2, 1, 4, 3, 8, 7, 5, 6, 9, 10

The assessment will consist of two partial exams and the delivery of the resolution of a problem through the comp

Bibliography

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B. Gustafson, H-O. Kreiss and J. Oliger, Time Dependent Problems and Difference Methods, Wiley-Intersciences, (1996).

F. John, Partial Differential equations, vol. 1, Applied Math Sciences, Springer, (1978).

P.D. Lax, Hyperbolic systems of Conservation Laws and The Mathematical Theory of Shock Waves SIAM, 1973.

R.J. LeVeque, Finite Volume Methods for Hyperbolic problems, Cambridge University Press, 2002.

Y.Pinchover, J. Rubinstein, An Introduction to Partial Differential Equations, Cambridge 2005.

S. Salsa, Partial differential equations in action : from modelling to theory Springer, 2008.

G. Strang, Introduction to Applied Mathematics, Wellesley-Cambridge Press, (1986).

E.F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics: A practical Introduction, Springer-Verlag, 2009.

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Software

We leave full freedom to students to use the language that suits them best to do the numerical exercises of this c

Language list

Name	Group	Language	Semester	Turn
(TEm) Theory (master)	1	English	second semester	afternoon