

Algebraic Structures

Code: 100096
ECTS Credits: 9

2025/2026

| Degree | Type | Year |
|-------------|------|------|
| Mathematics | OB | 2 |

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Teaching groups languages

You can view this information at the [end](#) of this document.

Prerequisites

The previous academic requirements are the contents of the courses *Fonaments de les matemàtiques* and *Algebra Lineal* in the first year.

There is specially important the skill on the operations in arithmetic, permutation groups and matrices, and also the polynomial ring and the vector spaces as a model of algebraic structure.

Objectives and Contextualisation

The objectives of this subject are of two types: to achieve training in basic algebra and gaining knowledge and skills to manipulate abstract objects.

Among the training objectives, we highlight the following: correctly understand and use language and mathematical reasoning in general and algebraic reasoning in particular. Be able to make small demonstrations, develop meaning critical of mathematical statements, develop combative attitudes and creativity in the face of problems and, finally, learn to apply abstract concepts and results in concrete examples. Present reasoning or a problem in public, and develop agility to answer mathematical questions in a conversation.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Calculate the maximum common divisor and factorisation of whole numbers and polynomials.
3. Construct quotient groups and rings and finite bodies and operate within them.
4. Operate in some simple groups (such as cyclic, dihedral, symmetric and abelian).
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
7. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

The subject is organized in four parts:

I. Group Theory.

- Groups, subgroups and morphisms. Basic examples.
- Action of a group on a set.
- Lateral classes. Lagrange Theorem.
- Normal subgroups, quotient group.
- Isomorphism theorems.
- Free groups, generators and relations.
- Sylow's theorems.
- Solvable groups.

II. Commutative rings

- Rings, ideals and morphisms. Basic examples.
- The ring of polynomials.
- Quotients and isomorphism theorems.
- Maximal and prime ideals.
- Field of fractions of a domain.

III. Factorization.

- Domains of main ideals.
- Unique factorization domains.
- Gaussian lemma. Factorization in rings of polynomials.

IV. Finite fields.

- Fields, subfields and characteristic of a field.
- Existence and uniqueness of finite fields.

Activities and Methodology

| Title | Hours | ECTS | Learning Outcomes |
|---------------------|-------|------|-------------------|
| Type: Directed | | | |
| Directed | 16 | 0.64 | |
| Theory classes | 43 | 1.72 | |
| Type: Supervised | | | |
| Seminars | 14 | 0.56 | |
| Type: Autonomous | | | |
| Seminar preparation | 145 | 5.8 | |

This subject has three hours per week of theory classes, and one hour per week of problem classes, and, during the semester, eight seminar sessions, two hours each.

Students will have the lists of problems previously to be able to work before the problem classes. It is highly recommendable they do it. Not all problems will be solved in the blackboard at class, and we strongly recommend that students work on their own the rest of the problems and ask the teachers if they have questions.

There will be eight seminar sessions. In these sessions, the students will work under the supervision of the teacher. There will be some exercises related to the seminar sessions to be solved that will count for the final mark of the subject.

The time written in the table is approximate, and the students should adjust the time in terms of their necessity. In any case, this subject consists of the 30% of the credits of a usual semester, so it is recommendable a dedication of 12 to 14 hours per week, including the classes.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continuous Assessment Activities

| Title | Weighting | Hours | ECTS | Learning Outcomes |
|----------------------------------|-----------|-------|------|------------------------|
| Continuous evaluation activities | 20% | 1 | 0.04 | 2, 3, 1, 4, 8, 7, 6, 5 |
| Exams | 80% | 6 | 0.24 | 2, 3, 1, 4, 8, 7, 6, 5 |

20% of the grade of the course corresponds to the delivery of problems to the seminars (S).

40% of the grade, P1, corresponds to the one obtained in a first partial examen, to evaluate the theoretical and practical abilities of the subject.

40% of the grade, P1, corresponds to the one obtained in a second partial examen. In this exam the theoretical and practical knowledge of the subject will be evaluated.

The final grade of the course will be obtained from the formula $N = 0.20 \cdot S + 0.40 \cdot P + 0.40 \cdot F$. The course will be passed if the grade N is equal to or higher than 5.

The honours will be awarded based only the value of the grade N.

For those whom have obtained a grade $N < 5$, there will be a remedial exam. If we denote the grade obtained for this exam by R, and $N' = 0.20 \cdot S + 0.80 \cdot R$ is bigger or equal than 5, then the grade obtained in the course will be 5. If $N' < 5$, then the grade will be the maximum between N and N'.

In the unique evaluation option, there will be an exam on the same day of the second partial exam, that will consist of two parts, corresponding to each of the partial exams; moreover, and during the same day if it is possible, there will be a test (written or oral) corresponding to the seminars. The remedial exam, if it is necessary, will be as the rest of the class.

Bibliography

[1] R. Antoine, R. Camps, J. Moncasi. Introducció a l'àlgebra abstracta. Manuals de la UAB, Servei de

Publicacions de la UAB, no. 46, Bellaterra, 2007.

[2] F. Cedó, V. Gisin, Àlgebra bàsica, Manuals de la UAB, Servei de Publicacions de la UAB, no. 21, Bellaterra, 2007.

[3] David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd. Edition, Wiley, 2003.

[4] J.B. Fraleigh. A First course in abstract algebra. Pearson Education, 7th Edition, 2014. Review:

<https://www.maa.org/press/maa-reviews/abstract-algebra>

[5] T. W. Hungerford, Abstract Algebra, Brooks/Cole, 2013. Review:

<https://www.maa.org/press/maa-reviews/abstract-algebra-an-introduction>

Software

Sagemath and the included programs such as GAP can be used to do computations in most of the parts of this course.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this [link](#). To consult the language you will need to enter the CODE of the subject.

| Name | Group | Language | Semester | Turn |
|----------------------------|-------|----------|-----------------|---------------|
| (PAUL) Classroom practices | 1 | Catalan | second semester | morning-mixed |
| (PAUL) Classroom practices | 2 | Catalan | second semester | morning-mixed |
| (SEM) Seminars | 1 | Catalan | second semester | morning-mixed |
| (SEM) Seminars | 2 | Catalan | second semester | morning-mixed |
| (TE) Theory | 1 | Catalan | second semester | morning-mixed |