UAB
Universitat Autònoma de Barcelona

Differential Equations and Modelling II

Code: 100101
ECTS Credits: 6

2025/2026

Degree	Type	Year
Mathematics	OB	3

Contact

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Teachers

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

Mathematical analysis in one and several variables, Linear Algebra and a first course on Differential Equations and modeling.

Objectives and Contextualisation

This assignment is the second part of a course on fundamental differential equations. Similar to the assignment Differential Equations and Modeling I, this second part has both a theoretical (which will be explored in theory and problem-solving classes) and an applied (which will be introduced in theory classes and applied both in problem-solving and practice classes) components. It is important that the students understand how to apply the concepts of the quantitative theory of differential ordinary equations to problems and have a knowledge of the basic partial differential equations. In addition to applying a number of the well-established and well explored findings from Equations Differential and Modeling I, new techniques for investigating the named differential equations will also be introduced.

Competences

- Distinguish, when faced with a problem or situation, what is substantial from what is purely chance or circumstantial.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.

- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.

Learning Outcomes

1. Extract quantitative information about the solution to an ordinary differential equation, without the need to resolve it.
2. Know how to draw simple phase portraits of systems of planar differential equations.
3. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
4. Study the behaviour of the solutions to differential equation systems in accordance with the parameters defining them.

Content

There are three sections to this topic. The first one focuses specifically on planar autonomous systems and discusses the qualitative theory of ordinary differential equations. It serves as an introduction to a topic that will later be covered in greater detail in the *Dynamical systems course*. The second and third one, have continuity with the course *Partial differential equations* and are a first study of the most well-known partial differential equations.

1 Autonomous systems in the plane.

1.1. Autonomous systems in R^n. Geometric interpretation. Orbits' structure. First integrals. Invariant surfaces. Phase portraits and conjugation.

1.2. Integrable systems. Phase portrait of planar integrable systems: potential systems, Hamiltonian systems. The model of Lotka-Volterra. The frictionless pendulum model.

1.3. Non-integrable systems: qualitative analysis of equilibrium points, stability and Lyapunov functions, limit behavior of the orbits, Poincare-Bendixson theorem. Limit cycles. Criterion of Bendixson, criterion of Bendixon-Dulac. Models of ecology. Van der Pol system.

2 First order partial differential equations.

2.1. Introduction to partial differential equations(PDE).

2.2. Linear and quasi-linear PDE of first order. Method of characteristics.

3 Second order partial differential equations.

3.1. Initial and boundary condition problems. The heat equation. The case of a finite bar.

3.2. The Laplace's equation.

3.3. Variable separation and Fourier series.

3.4. The wave equation on a finite and on an infinite string. D'Alembert's formula.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Classes of problems	15	0.6	
Classes of theory	30	1.2	
Type: Supervised			
Practical classes	6	0.24	
Type: Autonomous			
Personal studies	88	3.52	

Three types of in-person activities will be conducted: theoretical classes, problem-solving classes, and practical classes.

- In the theoretical classes, the professor will introduce the study topics, explain the material, and include examples. The professor will provide theoretical material on the virtual campus, which students can use as support, keeping in mind that this is only part of the material explained. Under no circumstances does this material replace the reference literature of the course.
- In the problem-solving classes, the professor will present solutions to some representative problems. The problem lists may include more problems than those solved in class.
- In the practical classes, three topics of the course will be studied in more detail. It will be mandatory to submit in writing problems related to it. This submission will be done in pairs (a group of three may be allowed if required by the total number of students in the course)

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final test	90%	4	0.16	4, 1, 3, 2
First test	45%	3	0.12	4, 1, 3, 2
Practical exercices	10%	0	0	4, 1, 3, 2
Second test	45%	4	0.16	4, 1, 3, 2
Single examination (SE) - for the students who has requested a single evaluation method (4h)	90%	0	0	4, 1, 3, 2

Continuous assessment /Ongoing evaluation:

- A mandatory delivery of practical assignments (seminars). We denote *PR* as the grade out of 10 obtained from these submissions. This is a non-recoverable activity.

- A first midterm exam covering theory and problem-solving. We denote *E1* as the grade out of 10 obtained in this exam.

- A second midterm exam at the end of the course, also covering theory and problem-solving. We denote *E2* as the grade out of 10 obtained in this exam.

- Additionally, during the problem-solving classes held between the first and second midterms, a couple of exercises may be offered to be done jointly in two-person groups and submitted through the CV. These problems will have a maximum score of 0.25 each, which will be added to E2 only if E2 >= 3.5.

Unique assessment:

- A student who has requested a single evaluation method is required to take a final test on the same day as their peers who are taking the second part of the exam. This test will be a single examination (*SE*) that includes all of the E1 and E2 evaluation criteria and is weighted at 10. The practice reports that were provided to him/her on the virtual campus will be turned in when the exam is over, and this delivery's note will be qualified with *PR*. As with the other students, it is essential that PR>=4 and recovery is NOT possible. In the same way, SE must be greater than (or equal to) 3.5, and if it falls short, the student has the chance to pass SE by passing a comprehensive exam covering the entire course (see the next paragraph talking about this).

Recovering exam for both cases above:

- Exam covering the 90% of the course with a grade of FE, also about 10. In no circumstance is the practice's note recoverable. If FE<3.5, the entire course note will be NC=FE. If FE>=3.5 and PR<4, the entire curse note will be NC=min(FE, PR).

Course qualification:

- For students receiving ongoing evaluation. The final course grade (NC) applies if 1) PR >= 4 and 2) min(E1,E2)>3.5, unless ((E1+E2) /2) >= 5. The formula will be: NC = (4.5E1+4.5E2+PR) /10.
- For the single evaluation students. The final course grade (NC), applies if PR >=4 and SE>=3.5, using the formula NC = (9SE+ PR) /10.
- In either of the two aforementioned scenarios, 90% of the grade (i.e. the portion not related to PR) may be recovered via the final exam (FE). In this situation, the final course grade will be NC = (9FE+PR) /10 if PR>=4 and FE>=3.5.
- Possibility of improving the grade: Additionally, students who choose to can attempt to improve the grade for the 90% that does not correspond to practicals. The exam will be a final exam (the same exam as the FE exam) and, therefore, will be held on the same day as the assigned recovery exam for the course. It should be noted that when a student attempting to improve their grade submits the exam (FE), they are forfeiting the grade they are trying to improve (i.e., they forfeit their grade ((E1+E2)/2) in the case of continuous assessment or their SE grade in the case of single assessment).
- The possible grade with honors will be awarded based on the course syllabus. Honors grades will not be awarded due to a very good FE grade (regardless of whether the person has taken the exam to improve their grade or for recovery).
- A student will be considered not assessable (***NO evaluable)*** for the course if they have participated in assessment activities representing less than 50% of the total grade, according to the established weighting.

Bibliography

For the course's first section, the required reading list will be as follows:

- "Ecuaciones diferenciales, sistemas dinámicos y àlgebra lineal", Morris W. Hirsch, Stephen Smale, Alianza Universidad Textos, Madrid, 1983.
- "Equaçoes Diferenciais Ordinarias", J. Sotomayor.
- "Qualitative Theory of Planar Differential Systems", Freddy Dmortier, Jaume Llibre, Joan C. Artés, Universitext, Springer, 2006.

Regarding the second and third topics:

- "Primer curso de ECUACIONES EN DERIVADAS PARCIALES", Ireneo Peral, UAM, Madrid, 1995. (pdf accessible at the Peral professor's web)
- "EDP, um curso de graduaçao", Valéria Iório, IMPA, Brasil, 2001.
- "Partial Differential Equations Vol I", M.E. Taylor, Applied Mathematical Sciences, 2011.

As an auxiliary bibliography for the three topics, the following is suggested:

- "Models amb Equacions Diferencials", R. Martínez. Materials de la UAB no. 149. Bellaterra, 2004
- "Equaçoes Diferenciais: Teoria Qualitativa", L. Barreira i C. Valls, IST Press Lisboa 2010.
- "Ecuaciones Diferenciales y Cálculo Variacional ", Lev Elsgoltz, Mir, Moscou, 1983.
- "Apunts d'Equacions Diferencials", d'en Francesc Mañosas, UAB (available through the virtual campus)
- "Ecuaciones diferenciales", V. Jimenez. Serie: enseñanza. Universidad de Murcia, 2000.
- "Análise de Fourier e equaçoesdiferenciais parciais", Djaro guedes de Figueiredo, IMPA, Brasil, 2000.
- "Càlcul Infinitesimal amb Mètodes Numèrics iAplicacions", C. Perelló. Enciclopèdia Catalana, 1994.
- "Ecuaciones Diferenciales y Problemas con Valores en la Frontera", E. Boyce, y R.C. Di Prima, Ed. Limusa, México, 1967.
- "Partial Differential Equations, An Introduction", Walter Strauss, Wiley, New York, 1992.
- "Elliptic partial differential equations of second order", *David Gilbarg,* Berlin: springer, 1977.

Software

For the practical classes, we can use SAGE.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(PAUL) Classroom practices	2	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	morning-mixed
(SEM) Seminars	2	Catalan	second semester	morning-mixed
(TE) Theory	1	Catalan	second semester	morning-mixed