Complex and Fourier Analysis

Code: 100103
ECTS Credits: 6

2025/2026

Degree	Type	Year
Mathematics	OB	3

Contact

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Teachers

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

It is a third-year subject; therefore, the students already have a certain mathematical background necessary to follow it. Although it will be quite self-contained, certain prior knowledge is essential. For example, the theory of series and power series and the improper integrals of Mathematical Analysis and the differential calculus in several variables, including Lebesgue integration. Although some aspects of complex numbers have already been seen in other courses, they will be repeated here to facilitate students' learning.

Objectives and Contextualisation

The course aims to provide students with a solid understanding of complex-variable functions and their applications. Specifically, it aims to:

- Master the fundamental concepts of complex analysis, such as holomorphic functions, the Cauchy-Riemann equations, and power series.
- Apply Cauchy's theory and its main results to the computation of complex integrals and the study of properties of holomorphic functions.
- Identify and analyze singularities, use the residue theorem, and apply it to integral evaluation.
- Work with harmonic functions and understand their relationship to holomorphic functions in simply connected domains.
- Understand conformal mapping techniques, including homographic transformations and results such as the Riemann mapping theorem.

- Foster critical thinking, mathematical rigor, and clarity of expression, as well as the ability to reason and solve problems effectively.
- Use computational and visualization tools to experiment with and reinforce understanding of the course material.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Calculate and reproduce certain mathematical routines and processes with agility.
- Recognise the presence of Mathematics in other disciplines.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Comfortably deal with the calculation of residues and its applications
4. Contrast acquired theoretical and practical knowledge.
5. Handle with ease homographic transformations and conformal representation.
6. Know how to calculate Fourier periodic function coefficients and their possible immediate applications for calculating sums of series.
7. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
8. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
9. Understand Fourier and Laplace transforms of elemental functions and their application to the resolution of differential equations.
10. Understand the basic results and fundamental properties of holomorphic functions and Cauchy theory.

Content

1. Preliminaries. Complex numbers. Power series. Holomorphic functions. Cauchy-Riemann equations.
2. Local theory of Cauchy. Complex line integrals. Cauchy-Goursat theorem and local Cauchy theorem. Holomorphy and analyticity. Zeros of holomorphic functions. Index of a closed curve. Cauchy's integral formula. Analytic continuation. Cauchy inequalities, Liouville's theorem, and the Fundamental Theorem of Algebra. Maximum modulus principle. Schwarz's lemma.

3. Singularities. Laurent series. Classification of isolated singularities. Residue theorem and applications. Argument principle and Rouché's theorem.
4. Harmonic functions and basic properties. Harmonic functions in a disc. Dirichlet problem.
5. Transforms. Fourier transform. Laplace transform. Basic properties. Applications to the solution of equations.
6. Convergence in the space of holomorphic functions. Weierstrass theorem. Hurwitz's theorem. Riemann's conformal mapping theorem.

NOTE: Either chapter 5 or 6 will be covered, depending on the time available and in order to make the course more complete. In the last two years, topic 6 has been taught, in line with the new course planned in the upcoming curriculum.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exercices	14	0.56	2, 10, 9, 4, 1, 3, 5, 6
Maths seminar	6	0.24	2, 10, 9, 4, 1, 3, 5, 8, 7, 6
Theory	28	1.12	2, 10, 9, 1, 3, 5, 8, 7, 6
Type: Autonomous			
Studying time	88	3.52	2, 10, 9, 4, 1, 3, 5, 6

The subject has two hours of theory per week. They will be taught in the traditional way with chalk and blackboard. In the theory where the concepts will be broken down and the important results (theorems) that underpin the theory we are introducing will be stated.

We will demonstrate theorems and solving methods through examples and exercises.

The student will receive lists of exercises and problems that we will work on in the weekly problem class. Previously, during your off-site activity, you will have read and thought about the proposed exercises and problems. In this way, their participation in the classroom can be guaranteed and the assimilation of procedural content will be facilitated.

There will be three seminar sessions, each lasting two hours. The students will have material previously placed on the Virtual Campus that they must have studied. In the first two sessions there will be a (short) first part where the teacher will add some details about the content of the practice. Then the students will get to work on a list of activities. Practices can be done in pairs, which seems to help them a lot. The third session of the seminars will be assessable. The planned topics are a more in-depth study of Möbius transformations and more applications of the residue theorem in the calculation of definite integrals. The evaluation will deal with these topics.

The Virtual Campus will be the means of communication between teachers and students. It will be important to consult it every day.

The students will have a tutoring and counseling service in the office. It is recommended to use this aid for monitoring the course.

Note: 15 minutes of a class will be reserved within the calendar established by center/degree for completion by students in the evaluation surveys of the teaching staff's performance and evaluation of the subject/module .

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	40	4	0.16	2, 10, 4, 1, 3, 8, 7
Maths Seminars	20	2	0.08	10, 9, 3, 5, 6
Recovery exam	80	4	0.16	2, 10, 9, 4, 1, 3, 5, 8, 7, 6
Second partial exam	40	4	0.16	10, 9, 3, 8, 7, 6

Learning mathematics is a complex process. It requires a maturation that develops over the course. Often, some results from the beginning of the theory are only fully understood much later in the semester. This highlights the difficulty of assessments.

Two written midterm exams will be held during the semester, mainly focused on problem-solving, but also including a theoretical part. These will be graded as P1 and P2, respectively. The exam grade will be the arithmetic mean:
$E = (P1 + P2)/2$
The seminar component will provide a grade S worth up to 20%.
The preliminary grade for continuous assessment will be calculated as:
$QP = 0.8 \cdot E + 0.2 \cdot S$
According to this formula, if a student obtains:
$E < 3.75$
(the average of the midterms), they will not be able to pass the course. This does not prevent the student from taking the resit exam, but from computing optional activities into the final mark.

If deemed appropriate by the instructor, interviews with students may be requested to refine the grading. Other optional activities may be offered during the semester to improve the final grade, such as forum participation, individualized tasks, or assignments. These improvements will only apply if the student has achieved an average $E \geq 3.75$. For example, if a project is proposed with a weight of 10%, the continuous assessment grade will be:
$QC = 0.9 \cdot QP + 0.1 \cdot \max(QP, T)$, where T is the grade of the project.
If $QC \geq 5$, the course will be considered passed.

Otherwise, the student may take a resit exam, receiving grades R1 and R2 corresponding to the retakes of each midterm. The retake average will be:
$R = (\max(P1, R1) + \max(P2, R2))/2$
and the final retake grade:
$QR = \min(0.8 \cdot R + 0.2 \cdot S, 5)$
meaning that the maximum achievable grade through resit is 5.

The final course grade will always be:
$QF = \max(QC, QR)$

Honors distinctions will be awarded in accordance with current regulations and only once all assessments have been completed.

If a student attends only one assessment activity, their final grade will be "Not assessable".

SINGLE ASSESSMENT:

Students who, for well-justified reasons, cannot follow the continuous assessment model, may opt for single assessment. This must be formally requested under the conditions established by the degree program. Accepted students will take the exam along with the resit exam, with two additional questions on seminar content. This will yield a grade S.
Their final grade will be:
QU = 0.8·R + 0.2·S

If QU is above 3.5 but below 5, the student will be entitled to a resit under the same conditions, with a maximum grade of 5.
If QU does not exceed 3.5, that will be the final grade assigned.

Bibliography

Bibliografia bàsica:

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2. L. Ahlfors, *Complex Analysis*, McGraw-Hill, 3a edició, 1979.
(És una referència clàssica que, amb un format reduït, tracta molts temes de forma rigorosa).

3. J. Conway, *Functions of One Complex Variable*, 2a edició, Springer-Verlag, 1978.
(Abarca molt més que el curs i conté molts problemes).

4. J. P. D'Angelo, *An Introduction to Complex Analysis and Geometry*, AMS, 2010.
(És una introducció de nivell més elemental que els dos anteriors).

5. B. Davis, *Transforms and Their Applications*, 3a edició, Springer, 2001.
(Serveix com a inici i aprofundiment en l'estudi de les transformacions integrals).

6. M. C. Pereyra i L. A. Ward, *Harmonic Analysis: From Fourier to Wavelets*, AMS, 2012.
(Curs força complet d'anàlisi harmònica).

Bibliografia complementària:

1. J. Bruna i J. Cufí, *Anàlisi Complexa*, Manuals UAB 49, 2008.

2. R. Burckel, *Introduction to Classical Complex Analysis*, vol. I, Academic Press, 1979.

3. W. Rudin, *Análisis Real y Complejo*, Alhambra, 1979.

4. S. Saks i A. Zygmund, *Fonctions Analytiques*, Masson et Cie, 1970.

5. E. Stein i R. Shakarchi, *Complex Analysis*, Princeton University Press, 2003.

Software

There are no computer practice classes in the subject, so no study of computer programs will be done. Despite this, it will be recommended to use mathematical manipulation programs such as Sagemath, Maxima or Wolfram Alpha, which can be very useful.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(PAUL) Classroom practices	2	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	morning-mixed
(SEM) Seminars	2	Catalan	second semester	morning-mixed
(TE) Theory	1	Catalan	second semester	morning-mixed