UAB
Universitat Autònoma de Barcelona

Differential Geometry

Code: 100107
ECTS Credits: 12

2025/2026

Degree	Type	Year
Mathematics	OB	3

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

It will be convenient to have a good prior knowledge of Calculus in several variables (differentiability, diffeomorphism, inverse function theorem and integration), Algebra and linear geometry (Euclidean spaces, bilinear forms, diagonalization of self-adjoint endomorphisms), and Topology (open, connectedness, homeomorphism).

Results from the courses of Differential Equations (theorem of existence and uniqueness of solutions, and the particular case of systems of linear differential equations), and Fundamentals of Mathematics (symmetric group) will also be used.

Objectives and Contextualisation

Differential Geometry is key to understanding the world around us. It serves as the foundation of theoretical physics, giving it the rigorous framework necessary for the formalization of some of its theories, from Maxwell's classical Electrodynamics to Einstein's Restricted Relativity and General Relativity. Geometry also teaches us how we can think in larger dimensions than the three-dimensional space in which we live, and opens our horizon of thinking.

The fundamental objective of this subject is to understand how the linear geometric objects that are the affine subspaces can be made more flexible, into curved or twisted geometric objects.

We will first study the parameterized curves of the Euclidean space, describing the different invariants that we can define according to the dimension in which they are developed: curvature in any dimension, curvature with sign in dimension 2, and finally torsion and Frenet reference in dimension 3. Secondly we will explain how we can generalize curves, thought of as one-dimensional non-linear objects, to the higher dimension. It will be the introduction of the concept of submanifolds that will need to explore the local structure of injective or surjective infinitesimally differentiable applications (immersions and submersions). This new concept of submanifold will be indispensable when understanding the content that will be presented in the 4th year courses Topology of Manifolds and Riemannian Geometry. In a third part, we will deeply study this new concept in the very particular case that corresponds to our everyday reality. It will be the notion of regular surfaces, for which we will define the notion of first and second fundamental form, and of curvature. We will also study how curves behave on these geometric objects, making the relationship between the invariants described in the first part of this course and this third. We will also introduce special families of curves on surfaces such as geodesics, lines of curvature or asymptotic lines. In the last part, we will present the notion of differential form. Here, we will again make a leap in the degree of abstraction to define the notion of integration of differential forms in the submanifolds described in the second part. The reward for this theoretical work will be Stokes theorem which will be the conclusion of the route proposed in this subject.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Recognise the presence of Mathematics in other disciplines.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Apply the integrals of line and surface to recognise some global properties of crooked and surfaces.
4. Know pose and resolve curvilinear and integral integrals of surface.
5. Recognise the nature of the points of an R3 curve Calculate curvature and torsion.
6. Recognise the nature of the points of an R3 surface. Calculate Gauss curvature, mean curvature and principal curvatures.
7. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.

8. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
9. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
10. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
11. Topologically recognise compact spaces and their classification.
12. Understand the applications of vector calculus and differential geometry calculus to physics problems.
13. Use some kind of scientific program to make computations and display surfaces.

Content

1. Parameterized curves
1.1. Parameterized Rn curves: definitions, length, parameter change, arc parameter, curvature and normal vector.
1.2. Geometry of the curves of R2: curvature with sign, singular points.
1.3. Geometry of R3 curves: torsion and Frenet's formulas, local canonical form, fundamental theorem of the local theory, torsion and Frenet's formulas in the general case.

2. Submanifolds
2.1. Local structure of immersions and submersions: definitions, local structure theorems.
2.2. Subvarieties: definition, characterizations, local parameterizations, tangent space, differentiable applications.

3. Regular surfaces
3.1. First fundamental form: definition, calculation of length, area, isometries.

3.2. Second fundamental form: orientation, definition, principal, Gaussian, and average curvatures, local expression, normal curvature, curvature and asymptotic lines.
3.3. Gauss's Egregium theorem: Christoffel's symbols, expression of Gauss's curvature.
3.4. Geodesics: vector field along a curve, covariant derivative, parallel transport, geodesics, geodesic curvature.

4. Differential Forms
4.1. Rn vector fields: definitions, integral curves.
4.2. Multilinear algebra: multilinear forms, outer product, evaluation via the determinant, decomposition.
4.3. Differential forms on Rn: definition, external differential, pullback, volume form on Rn.
4.4. Bounded submanifolds: definitions, vector fields, differential forms and orientation.
4.5. Integration and Stokes theorem: Integration of differential forms, Stokes theorem.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem solving classes	30	1.2	3, 12, 9, 7, 5, 6, 11, 4, 13
Theory classes	45	1.8	3, 12, 9, 7, 5, 6, 11, 4, 13

Type: Supervised			
Seminars	28	1.12	3, 12, 9, 7, 5, 6, 11, 4, 13
Type: Autonomous			
Autonomous study	178	7.12	3, 12, 9, 7, 5, 6, 11, 4, 13

The course has three hours of theory classes, two of problems, and two of seminars/practices per week for 15 weeks of the course.

The theory classes will introduce the fundamental concepts and will explain the topics of the program to encourage the students to ask questions and actively participate in the class.

The classes of problems will be solved in exercises and questions will be analyzed that clarify and develop the notions introduced to the theory classes. This work is hard to finish by mitigating the explanations given by the teacher to the Pissarra and the active participation of the students in the discussion of the different arguments used in order to solve the problems, which the student will have to think about before during study hours. , individually or in a group.

The seminar sessions are mainly dedicated to developing some topics on the part of the student, but also to delve into the questions raised in the theory class in an autonomous way. During the session, the students will give answers to the questions raised by the teachers who will resolve specific doubts and comment on the most important aspects of the topic to be developed. At the end of each session, the teachers will inform the students if they have to deliver a written report with the resolution of any of the questions formulated.

Apart the students are expected to deliver personal exercises online through the ACME platform.

Note: 15 minutes of a class will be reserved, according to the calendar established by the center/degree, for the completion by the students of the surveys for the evaluation of the teacher's performance and the evaluation of the subject /module.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Delivery of solved problems P	25%	10	0.4	3, 2, 12, 10, 9, 8, 7, 5, 6, 11, 4, 13
Exam E1	30%	3	0.12	3, 2, 12, 10, 9, 7, 5, 6, 11, 4
Exam E2	45%	3	0.12	3, 2, 12, 10, 9, 7, 5, 6, 11, 4
Recovery exam ER	75%	3	0.12	3, 1, 12, 10, 9, 7, 5, 6, 11, 4

The final grade (continuous assessment) of the subject will be calculated as follows:

- 30% of the grade will correspond to the completion of a partial exam.
- 25% of the grade will correspond to assignments of problems and/or practices. In particular, attendance at seminars is mandatory.
- 45% of the grade will correspond to the completion of a final exam.

The student passes the subject if his final grade is greater than or equal to 5.

The recovery exam replaces both the final and mid-term exams. A student taking the recovery exam and (together with the seminar and ACME's note) passing the course will be awarded the final note of 5.
To be able to attend the recovery, the student must have previously been assessed for continuous assessment activities that are equivalent to at least 2/3 of the final grade.

Honors will be awarded after the final exam. These honor registrations will be definitive. It is considered that the student is present for the course evaluation if he has participated in evaluation activities that exceed 50% of the total. Otherwise, your rating will be Non-Evaluable.

The single assessment of the subject will consist of the following assessment activities:

- Taking the final exam, for 45% of the grade.
- Delivery on the day of the final exam of the assignments requested in the seminars, for 25% of the final grade. In particular, attendance at seminars is mandatory.
- Taking an oral exam, for 30% of the grade.

Bibliography

Manfredo P. do Carmo. Geometría diferencial de curvas y superfícies. Alianza Editorial (1990).

Theodore Shifrin. Differential Geometry: A First Course in Curves and Surfaces. Accesible a la pàgina de l'autor (2021).

Sebastián Montiel y Antonio Ros. Curvas y superfícies. Proyecto Sur (1998).

Michael Spivak. Cálculo en Variedades. Ed. Reverté (1970).

Victor A. Toponogov. Differential Geometry of Curves and Surfaces. Birkhäuser (2006).

Shoshichi Kobayashi. Differential Geometry of Curves and Surfaces. Springer (2019).

Software

SageMath software will be used in some of the seminars.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn

(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(PAUL) Classroom practices	2	Catalan	second semester	morning-mixed
(PLAB) Practical laboratories	1	Spanish	second semester	morning-mixed
(PLAB) Practical laboratories	2	Spanish	second semester	morning-mixed
(SEM) Seminars	1	Spanish	second semester	morning-mixed
(SEM) Seminars	2	Spanish	second semester	morning-mixed
(TE) Theory	1	Catalan	second semester	morning-mixed