UAB
Universitat Autònoma de Barcelona

Harmonic Analysis

Code: 100111
ECTS Credits: 6

2025/2026

Degree	Type	Year
Mathematics	OT	4

Contact

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Teachers

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

The first and second year Analysis courses of the mathematics degree. It is also useful to have followed the cour

Objectives and Contextualisation

The course aims to provide an introduction to Harmonic Analysis in euclidean spaces. In particular, it focuses on

The topic chosen for the last part of this year's course will be an introduction to Restriction Theory, a significant a

A part from the harmonic analytic interest of the contents, the course will also expose the students to several tech

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
3. Effectively use bibliographies and electronic resources to obtain information.
4. Formulate conjectures and devise strategies to confirm or reject said conjectures
5. Generate innovative and competitive proposals for research and professional activities.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
10. Understand and know how to reproduce basic results in relation to the Hilbert transform.

Content

1. Convolutions and approximate identities.
2. Fourier transform of integrable and L^2 functions. Fourier inversion and uncertainty principle.
3. Riesz-Thorin interpolation theorem and applications to the Fourier transform. Young's and Hausdorff-Young inequality.
4. Introduction to the theory of tempered distribution: motivation, Schwartz functions, Schwartz seminorms, tempered distributions, basic operation with tempered distributions, Fourier transform of tempered distributions.
5. Introduction to the restriction theory for the Fourier transform: stationary phase principle, statement of the restriction conjecture, Tomas-Stein theorem, and the Knapp example. If time permits, we will also explore some connections between the restriction conjecture and other important problems, such as the Kakeya conjecture.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Directed	30	1.2	5, 8, 6
Type: Supervised			
Supervised	20	0.8	10, 5, 8, 6
Type: Autonomous			
Autonomous	85	3.4	10, 5, 8, 6

The course will follow the standard methodology in Mathematics. In particular, there will be blackboard classes with discussion of definitions, examples and Theorems. There will also be problem sessions: students will be provided with exercise lists, that will be uploaded on the Campus Virtual of the course, and that will be partially solved and discussed in class.

Formally, they are 30 and 20 hours respectively, but in practice, we will do 50 hours all mixed together.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final exam	30%	2	0.08	1, 2, 10, 4, 5, 9, 8, 6, 7, 3
Homework exercises	25%	9	0.36	1, 2, 10, 4, 5, 9, 8, 6, 7, 3
Partial exam	35%	2	0.08	1, 2, 10, 4, 5, 9, 8, 6, 7, 3
Seminar activity	10%	2	0.08	1, 2, 9, 8, 6

The evaluation of the exam is based on:

a) A mid-term exam P1, on the first half of the course.

b) Submission of two sets of exercises, with equal weight and total grade LL. The exercises are to be completed at home and uploaded to the Campus Virtual.

c) A one-hour long exam on the seminar activities S.

d) A final exam P2 on the theory of the whole course. The exam will consists of questions chosen from a list of 10 to 15 questions that the professor is going to make available to the students on the Campus Virtual.

Students who take part in the partial exam a) may choose not to take the final exam d). In this case, the grade is determined by only a), b), and c), *but cannot be higher than 7/10*. In particular, the final grade is going to be

NF1 = min{0.5*P1 + 0.35*LL + 0.15*S, 7 }.

For students taking part in the final exam d), the final grade is going to be the maximum between NF1 and NF2 = 0.35*P1 + 0.25*LL + 0.1*S + 0.3*P2.

In part particular, *in no case the final grade is going to be lower than NF1.*

If both NF1 and NF2 are lower than 5, or the student wants to imporve their grade, they can take part in a resit exam whose grade R is going to determine the final grade

NF3 = 0.25*LL + 0.1*S + 0.65*R.

The grade for the assignments and the seminars cannot be retaken.

Single evaluation: see catalan version.

Bibliography

The professor will upload lecture notes on the Campus Virtual of the course. The lecture notes will be mainly based on parts of the following references:

1. E. Stein and R. Shakarchi, "Fourier Analysis, an introduction", Princeton Lectures in Analysis, Priceton Univresity Press 2007.

2. E. Stein and G. Weiss, "Introduction to Fourier analysis on Euclidean spaces", Princeton Mathematical Series, 1971.

3. L. Grafakos, "Fundamentals of Fourier Analysis", Springer, 2024.

4. L. Grafakos, "Classical Fourier Analysis. 3rd ed.", Springer, 2014.

5. T. Wolff, "Lectures on harmonic analysis. Edited by Izabella Łaba and Carol Shubin." University Lecture Series 29. 2003.

Some complementary books are:

1. E. Stein, "Harmonic analysis: Real-variable methods, orthogonality, and oscillatory integrals. With the assistance of Timothy S. Murphy." Princeton Mathematical Series. 43. Princeton, NJ: Princeton University Press, 1993.

2. R. Strichartz, "A Guide to Distribution Theory and Fourier Transforms". CRC Press, Boca Ratón, FL, 1994.

3. A. H. Zemanian, "Distribution Theory and Transform Analysis: An Introduction to Generalized Functions, with Applications, reprint edition". Dover Publications, New York, 1987.

4. C. Demeter, "Fourier restriction, decoupling, and applications."Cambridge University Press, 2020.

Software

None.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	afternoon
(TE) Theory	1	Catalan	second semester	morning-mixed