UAB
Universitat Autònoma de Barcelona

Commutative Algebra

Code: 100112
ECTS Credits: 6

2025/2026

Degree	Type	Year
Mathematics	OT	4

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

The knowlege of all the compulsory courses in algebra, specially the course *Estructures Algebraiques*.

Objectives and Contextualisation

The aim of this course is to give an introduction to the basic tools of the theory of commutative rings while using the category and functor class.

This means working on the basic concepts of categories and functors, also of rings, the structure of their ideals and the modules on it, delving into specific topics of each of these aspects.

At the end of the course, the student is expected to know the basic concepts of category and functor theory. General constructions on rings and modules, chain conditions, and some ideas about the prime spectrum of commutative rings. From here and depending on the topics in which you have gone deeper during the course, the objectives to be achieved may vary.

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Demonstrate a high capacity for abstraction.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.

- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
3. Effectively use bibliographies and electronic resources to obtain information.
4. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
5. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
6. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
7. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
8. Understand in-depth demonstrations of some theorems of advanced algebra and assimilate the definition of new algebraic structures and constructions, relating them with other knowledge and deducing their properties.
9. Use algebraic tools in different fields.

Content

Broadly speaking, the course will be structured following the contents of the classic reference book "M.F. Atiyah and I.G. Macdonald, Introducción al Álgebra Conmutativa" but emphasizing more the use of Category Theory language and also in the examples. We will also consider Dummit and Foote's book "Abstract Algebra"

The topics to be discussed will be

1. First approach to the language of categories and functors.

2. Categories of rings and categories of modules on them. Basic results. The Hom functor and the tensor product.

3. Free modules and module presentations. Classification of finitely generated modules over a domain of principal ideals.

4. Zorn's Lemma and chain conditions. Noetherianity and artinianity and its applications in ring and module

5. Location. Spectrum of a ring.

6. Artinian rings and Noetherian rings. Hopkins theorem.

7.Integral extensions. Noether's Normalization and Hilbert's Nullstellensatz.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes

Type: Directed		
Theory classes	30	1.2
Type: Supervised		
Problem classes	15	0.6
Seminars	6	0.24
Type: Autonomous		
Solving problems	60	2.4
Study of the theory	36	1.44

This course has two hours of theory per week. The recommended bibliography is interesting; sometimes during the course the students should complement and complete the content of the lectures using this bibliography.

There are problem classes (one hour per week). Every student should present the solutions of some lists of problems on the blackboard or in paper to the lecturer. To solve questions about the course the student can approach the lecturer during the class or in the office (during office hours). The solution of these problems will be based in general on the theory: the theorems and their proofs.

Seminars will be dedicated to develop collaborative problem-solving techniques.

The course has a web "campus virtual" where the lecturer will post the problems list and other relevant information about the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Mid-term exam	40%	3	0.12	8, 2, 7, 6, 4, 5
Problems presented at the problem class	20%	0	0	1, 6, 4, 3, 9
Seminar attendance	10%	0	0	1, 2, 6, 4, 5, 3, 9
Solved problems	30%	0	0	1, 2, 6, 4, 3, 9

The evaluation of the course will be a 60% of continued evaluation, and a 40% of the midterm exam.

- The continued evaluation will consist of the presentation of solutions of problems at the problem class, and also written presentation of some selected problems at the end of the course (solved individually or in group), the attendance to the seminars and the presentation of solutions of the exercises proposed in the seminars.

- The mark of the exams will be obtained from the midterm exam.

The "matrícules d'honor" will be decided taken into account the results of the continued evaluation and the exams.

The "non-evaluable" qualification will be awarded to students who do not do, at least, 50 % of the evaluation activities.

Single assessment: Those who opt for single assessment will take a single exam where all course contents will be assessed. This exam will take place at the end of the course in a date to be agreed with the professor responsible of the subject.

In case of doubt about the interpretation of the evaluation method, the Catalan written version remains as the reference.

Bibliography

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P. M. Cohn, Algebra, vol 2. Second Ed. John Wiley and Sons, New York, 1989.

David S. Dummit and Richard M. Foote, Abstract Algebra, Third edition, John Wiley & Sons, 2004

D. Eisenbud, Commutative Algebra with a view toward Algebraic Geometry. Springer, New York, 2004.

B. Hartley, T. O . Hawkes, Rings, modules and linear algebra. Chapman and Hall, London 1983.

N. Jacobson, Basic Algebra I, Basic Algebra II. W. H. Freeman and Company, New York, 1989.

E. Kunz, Introduction to Commutative Algebra and Algebraic Geometry. Birkhäuser, New York, 2013.

S. Lang, Algebra. Aguilar, Madrid, 1977.

B. A. Magurn, An algebraic introduction to K-Theory, Encyclopedia of Mathematics and its applications, 87, Cambridge, 2002.

J.S. Milne, A Primer of Commutative Algebra, 2009.

O. Zariski, P. Samuel, Commutative Algebra I, II, Van Nostrand, Princeton (1958, 1960).

Software

There is an entire branch of commutative algebra devoted to developing computational methods. There is free software that allows you to work with ideal rings and modules, we will try to learn a little about some of these software.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Catalan	second semester	afternoon
(TE) Theory	1	Catalan	second semester	morning-mixed