

## Riemannian Geometry

Code: 100115  
ECTS Credits: 6

**2025/2026**

Degree	Type	Year
Mathematics	OT	4

### Contact

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### Teachers

Alejandro García Sanchez

### Teaching groups languages

You can view this information at the [end](#) of this document.

### Prerequisites

For good follow-up of the subject it is recommended a good assimilation of the concepts introduced in the subject Geometría Diferencial.

Analysis knowledge (Calculation in various variables and optimization), topology (Topology) and differential equations (Differential Equations and Modeling I) will also be used.

### Objectives and Contextualisation

A Riemannian manifold is a differentiable manifold equipped with a positive-definite inner product on the tangent space at each point. Riemannian geometry studies these structures and originally emerged as a generalization of the intrinsic geometry of surfaces. Later, it proved to be a fundamental tool for the formulation of classical mechanics and, especially, for the general theory of relativity. More recently, it has played a key role in the proof of the Poincaré conjecture.

The two central concepts in Riemannian geometry are curvature and geodesics. The main goal of the course is to understand, from a geometric perspective and as far as possible, the interplay between these two notions. In this regard, we will explore how curvature affects the behavior of geodesics and the topology of manifolds.

### Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.

- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Demonstrate a high capacity for abstraction.
- Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
- Effectively use bibliographies and electronic resources to obtain information.
- Formulate hypotheses and devise strategies to confirm or reject them.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

## Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Develop critical thinking and reasoning and know how to communicate it effectively, both in one's own languages and in a third language.
4. Devise demonstrations of mathematical results in the field of geometry and topology.
5. Effectively use bibliographies and electronic resources to obtain information.
6. Generate innovative and competitive proposals for research and professional activities.
7. Students must be capable of collecting and interpreting relevant data (usually within their area of study) in order to make statements that reflect social, scientific or ethical relevant issues.
8. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
9. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
10. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
11. Understand abstract language and in-depth demonstrations of some advanced theorems of geometry and topology.

## Content

1. Riemannian manifolds. Notion of Riemannian length and volume.
2. Connections. Geodesics. Exponential map and Gauss Lemma. The Hopf-Rinow theorem.
3. Curvature. Jacobi fields.
4. Hyperbolic geometry.
5. Other topics in Riemannian geometry.

## Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
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Type: Directed

Theoretical course	30	1.2	2, 11, 9
Tutorials	14	0.56	2, 11, 4, 9
Type: Supervised			
Seminars	6	0.24	2, 11, 4, 9, 8
Type: Autonomous			
Personal study	45	1.8	2, 11, 9, 5
Preparation and exhibition of works	16	0.64	2, 11, 9, 8, 5
Resolution of problems	30	1.2	2, 4, 9, 8, 5

The subject has two weekly hours of theory class and one of problems. In addition, throughout the course there will be three seminars of two hours each.

- In the *theory* classes, the fundamental notions of Riemannian geometry will be introduced and the most important results of the theory will be presented. Likewise, the necessary tools for the understanding and resolution of problems will be given.
- The *problems* classes will be deepened in the assimilation and the understanding of the concepts developed in the theoretical classes will be improved by solving theoretical problems and exercises designed to increase the students' dexterity in their own calculations. This work will be carried out through explanations made by the teacher on the board and the active participation of students in the discussion of the different arguments used to solve the problems. The lists of problems will be delivered to the students throughout the semester.
- The *seminars* will be devoted to deepening issues dealt with in theory classes and problems. Students will receive a script before each seminar will be held. During the session, they will have to work autonomously, although they can be consulted by the teachers. Subsequently, they will deliver the solution to the problems worked during the seminar. These solutions will be corrected by the teachers, giving rise to a part of the continuous evaluation note.

At the same time, each student will draw up a work on a chosen subject from a list proposed by the teachers. This work will be delivered in writing, as well as exposing yourself in class. The evaluation of both aspects (written and oral) will also be part of the continuous evaluation.

Individual tutorials, or in small groups, are foreseen for students who want it in the professor's office. In the end the student will have received the theory and problems classes, as well as the seminars, all the necessary information (both the statements and their demonstrations), to face the partial test such as the final test. This subject will also offer resources through the Virtual Campus.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

## Assessment

## Continuous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Delivery of problems	0,2	2	0.08	2, 11, 1, 4, 10, 9, 8, 5
Exam	0,3	2.5	0.1	2, 11, 1, 4, 9, 8, 5
Presentation of works	0,2	2	0.08	2, 11, 3, 6, 10, 9, 8, 7, 5
Recovery test	0,3	2.5	0.1	2, 11, 1, 4, 9, 8, 5

The evaluation of this subject will take into account the assimilation of the contents, as well as the work done during the course, and will be carried out in the form of a continuous evaluation.

The final grade will be obtained by weighted average between the score obtained in the partial exam module (40%), the problem delivery module (30%) and the presentation module of work (30%).

The possible license plates will be awarded based on the continuous evaluation note.

Students who had not passed the continuous evaluation, that is to say that they had not obtained a final grade equal or superior to five, or that they want to improve their mark, will have a final test of recovery. A student will be qualified as "Not Presented" if the weight of the assessment activities in which he has participated does not exceed 50% of the weight of the continuous assessment of the subject.

### Unique assessment

Students who have accepted the single assessment modality will have to take a final test which will consist of an exam. These tests will be carried out on the same day, time and place as the examination of the continuous assessment modality. When finished, you will hand in the required assignments and deliverables in the continuous assessment activities.

The student's grade will be the weighted average of the previous activities, where the exam will account for 40% of the grade and the assignments and assignments 60%.

If the final grade does not reach 5, the student has another opportunity to pass the subject through the remedial exam that will be held on the date set by the degree coordinator. The part of work and deliveries is not recoverable.

## Bibliography

- 1- Manfredo P. do Carmo, Riemannian Geometry. Birkhäuser, 1992.
- 2- Manfredo P. do Carmo, Geometría diferencial de curvas y superficies. Alianza Universidad, 1990.
- 3- S. Gallot, D. Hulin, J. Lafontaine, Riemannian Geometry. Springer-Verlag, 1990.
- 4- Joan Girbau, Geometria diferencial i relativitat. Manuals de la UAB, Servei de Publicacions de la U.A.B., 1993.
- 5- John M. Lee, Riemannian Manifolds: An introduction to curvature. Springer-Verlag, 1997.
- 6- M. Spivak, A Comprehensive Introduction to Differential Geometry. Publish or Perish Inc, 1979.
- 7- J. Cheeger, D. Ebin, Comparison Theorems in Riemannian Geometry. Noth Holland, 1975.

## Software

We will not use software in this course.

## Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this [link](#). To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	second semester	morning-mixed
(SEM) Seminars	1	Spanish	second semester	afternoon
(TE) Theory	1	Spanish	second semester	morning-mixed