UAB Universitat Autònoma de Barcelona

Dynamical Systems

Code: 100118
ECTS Credits: 6

2025/2026

Degree	Type	Year
Mathematics	OT	4

Contact

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

Ordinary differential equations: existence and uniqueness of solutions of the Cauchy problem.

Linear differential systems with constant coefficients.

Linear algebra: spaces and vector subspecies, diagonalization.

Objectives and Contextualisation

This course is an introduction to the modern theory of dynamical systems. The first objective is for students to become familiar with the abstract notion of a dynamical system and the basic concepts of this theory: stability, attractor, invariant sets, omega limits, etc. The second objective is to understand the local behavior of both discrete and continuous dynamical systems in the vicinity of an equilibrium point or a periodic orbit. This local behavior is based on the topological classification of linear systems in $\mathbb{R}^n$, both those derived from the flow of ordinary differential equations (continuous dynamical systems) and those arising from function iteration (discrete dynamical systems). Linear systems are very important because, on the one hand, they appear in the study of many relevant physical phenomena, and on the other hand, they represent the first approximation to more complex systems.

The qualitative theory of differential equations began with the work of Poincaré around 1880, in the context of his studies in celestial mechanics. It aims to understand properties of solutions without the need to solve the equations, among other reasons because exact solutions are only possible in exceptional cases. This qualitative approach, when combined with appropriate numerical methods, can in some cases be equivalent to having the solutions of the equation. The course will deepen the knowledge and study-introduced in previous subjects-of the qualitative theory of differential equations in higher-dimensional spaces, with an emphasis on the local structure of equilibrium points (both degenerate and non-degenerate) and the stability of periodic orbits.

The final objective of the course is to introduce techniques for understanding global discrete dynamics. The guiding thread will be a parametric family of discrete dynamical systems: unimodal maps, which (for certain parameter values) exhibit dynamics that naturally lead to the notion of chaos. For these systems, numerical approximation is not feasible, and new tools are required to understand their dynamics. Chaotic systems often appear in applications (e.g., weather prediction problems, electrical circuits, etc.).

Competences

- Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
- Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
- Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Understand and use mathematical language.

Learning Outcomes

1. Actively demonstrate high concern for quality when defending or presenting the conclusions of one's work.
2. Apply critical spirit and thoroughness to validate or reject both one's own arguments and those of others.
3. Know how to apply the dynamical tools described in theory lectures to describe processes governed by differential equations.
4. Know how to demonstrate the results of partial derivative equations and dynamical systems.
5. Know how to solve certain theoretical problems and be understand the existence of certain open problems in the theory of partial derivative equations and dynamical systems theory.
6. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
7. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
8. Students must develop the necessary learning skills to undertake further training with a high degree of autonomy.
9. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

1. Dynamical systems on topological spaces.

- Dynamical systems defined by differential equations and by difeomorphisms.
- Orbits; Critical points and periodic orbits.
- Invariant sets and limit sets.
- Attractors. Liapunov stability.
- Conjugation of dynamic systems.

2. Study of local dynamics, discrete and continuous in R^n.

- Phase portraits near equilibrim and regular points.
- Topological classification of continuous and discrete linear systems.
- Stability (Liapunov's Functions)
- Hartman theorems, of the stable variety and of the central variety.
- Periodic orbits: Application of Poincaré and stability.

3. Global dynamics in continuous systems.

- Ordinary differential equations in R2 (Theorem of Poincaré-Bendixon, Theorem of Bendixon-Dulac, Existence and unicity of limit cycles, ...)
- Ordinary differential equations in dimension greater than 2.

4. Global dynamics in discrete systems.

- Iteration in dimensions 1 and 2.
- Unimodal applications.
- Chaos Bernoulli's shift. Smale's Horseshoe.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem solving classes	14	0.56	
Seminars	6	0.24	
Theoretical lessons	29	1.16	
Type: Autonomous			
Exam Preparation	15	0.6	
Problem solving	42	1.68	
Study of the theoretical part	32	1.28	

The course includes two hours of theoretical classes and one hour of problem-solving classes per week. During the semester, there will also be three seminar sessions, each lasting two hours.

Schedules and classroom assignments must be consulted through the UAB's online platforms. A dedicated space for this course will be available on the Virtual Campus (VC) to provide materials and share information related to the classes.

Theoretical classes: The teaching staff will develop the topics of the syllabus in the indicated order. A bibliography and part of the supporting materials, if necessary, for both theory and problem-solving, will also be available to students on the VC.

Problem-solving classes: Problem sets will be available on the VC. Some of these problems will be worked on in class.

During the seminars, certain concepts will be explored in greater depth and developed by the students.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First midterm	40%	3	0.12	2, 5, 1, 9, 8, 7, 6, 3, 4
Second midterm	40%	3	0.12	5, 9, 3, 4
Seminars (3 activities)	20%	6	0.24	2, 1, 9, 8, 7, 3

Continuous assessment

The course is organized into the following components, each of which carries a specific weight in the final grade:

Seminars (SEM): Reports and the work assigned during the three seminar sessions will be evaluated.
First midterm (P1): Written exam scheduled for the middle of the semester.
Second midterm (P2): Written exam at the end of the semester.
If $N1 = 0.2SEM + 0.4*(P1 + P2)$ is greater than or equal to 5, then N1 will be the final grade for the course. If N1 is below 5, the student may take a resit exam (R), and the final grade will be $N2 = 0.2*SEM + 0.8*R$. To be eligible, the student must have participated in at least 66% of the assessed activities.

Single assessment

On the same day as the second midterm exam of the continuous assessment, students who have previously opted for the single assessment will submit the work assigned during the seminar sessions (SEM) and take a final exam (F) covering the entire syllabus. The final grade will be $N3 = 0.2*SEM + 0.8*F$. If $N3 < 5$, the same resit system as in the continuous assessment will apply.

Bibliography

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D.K. ARROWSMITH, C.M. PLACE, Dynamical Systems, differential equations, maps and chaotic behaviour, Chapman & Hall Mathematics, 1992.

R.L. DEVANEY, An introduction to chaotic dynamical systems, The Benjamin/Cummings Publishing Company, Inc., 1986.

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R.L. DEVANEY, A first course in chaotic dynamical systems, Theory and Experiment, Studies in Nonlinearity, 1992.

F. DUMORTIER, J.LLIBRE and J.C. ARTES, Qualitative Theory of Planar Differential Systems, Universitext, Springer-Verlag Berlin, 2006.

C. FERNANDEZ, F. j. VAZQUEZ, J. M. VEGAS, Ecuaciones diferenciales y en diferencias. Sistemas Dinámicso, Thomson 2003.

J. GUCKENHEIMER, P. HOLMES, Nonlinear oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1993.

M. HIRSCH, S. SMALE and R. DEVANEY, Differential Equations, Dynamical Systems and an Introduction to Chaos, Elsevier Academic Press, 2004.

M.C. IRWIN, Smooth Dynamical Systems, Advanced series in Nonlinear Dynamics, vol.17, World Scientific, 2001.

S. LYNCH, Dynamical Systems with Applications using MAPLE, Birkhäuser, 2000.

L. PERKO, Differential Equations and Dynamical Systems, Springer-Verlag, 1996.

C. ROBINSON, Dynamical Systems: Stability, Symbolic Dynamics and Chaos CRC Press, 1999.

J. L. ROMERO, C. GARCIA, Modelos y Sistemas Dinámicos, Univesidad de Cádiz, 1998.

J. SOTOMAYOR, Liçoes de equacoes diferenciais ordinárias, Projecto Euclides, Gráfica Editora Hamburg Ltda., 1979.

Software

The student will be able to use any of the programming languages that he has knowledge (C, Sagemath, Maxima, Maple, Mathematica, ...). Knowledge of some symbolic computing software will be useful.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	first semester	morning-mixed
(SEM) Seminars	1	Catalan	first semester	afternoon
(TE) Theory	1	Catalan	first semester	morning-mixed