

Multidimensional Distributions

Code: 104857 ECTS Credits: 6

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Degree	Туре	Year
Applied Statistics	ОВ	2

Contact

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Teachers

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Teaching groups languages

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Prerequisites

Elementary probability. Real random variables. Differential and integral calculation. Elemental algebra: vector spaces and determinants.

Objectives and Contextualisation

The probability distribution of a random vector (discrete or continuous) is the main objective of this course.

We analyze the principal characteristics of the joint distribution: the vector of means, the variance-covariance matrix, the marginal and conditional distributions, etc.

As a principal example we study the multidimensional normal distribution. It is an important continuous distribution, with applications in linear model theory, multivariate analysis and statistical decision theory.

Theory and calculations related to continuous multidimensional distributions, strongly depends on integral and differential calculus with functions of several variables. Having this in mind, we review the principal technics focusing on multivariate probability applications.

Learning Outcomes

1. KM10 (Knowledge) Describe the characteristics of the distribution and density functions of random variables.

Content

- 1. Random vectors. Multivariate distributions.
- 1.1 Introduction to bivariate distributions.
- 1.2 Discrete case. Joint and marginal probability functions.
- 1.3 Continuous case. Joint and marginal density functions.
- 1.4 Joint distribution function.
- 1.5 Multivariate distributions. The multinomial distribution.
- 1.6 Functions of two or more random variables.
- 2. Independence and conditioning.
- 2.1 Independence of random variables.
- 2.2 Conditional distributions: discrete case and absolutely continuous case.
- 3. Expectation and other numerical characteristics.
- 3.1 Expectation of a function of a random vector.
- 3.2 Covariance and correlation function.
- 3.3 Conditional expectation.
- 3.4 Moment generating function.
- 4. The multidimensional normal law. Distributions related to the normal law.
- 4.1 Study of the bivariate normal distribution.
- 4.2 The multivariate normal distribution.
- 4.3 Distributions related to the normal law: chi-square, Student's t, Fisher-Snedecor's F.
- 4.4 Fisher's theorem. Cochran's theorem.
- 5. Convergence of random variables and limit theorems in probability theory.
- 5.1 Chebyshev's inequality. Convergence in probability. Weak law of large numbers.
- 5.2 Almost sure convergence. Strong law of large numbers.
- 5.3 Convergence in distribution. Central limit theorem.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Exam preparation	20	0.8	
Theory lessons	25	1	
Type: Supervised			
Computer laboratory practical sessions (with Maxima and R) and seminars	12	0.48	
Practical lessons about problems and exercises	12	0.48	
Type: Autonomous			
Personal work (exercices and theory)	63	2.52	KM10, KM10

This course is semester-long and consists of two hours of theory and one hour of problem-solving per week of in-person instruction. In addition, there will be three seminar sessions of two hours each and three two-hour laboratory sessions.

The introduction of concepts in the theory classes is essential for students to understand and grasp the foundations of probability theory introduced in this course. Knowledge of the notions presented in the theory lectures, the statements of propositions and theorems, as well as examples of applications, is indispensable for students to be able to solve the problems presented in the practical sessions using similar methods. The structure of definition-theorem-proof-application will be followed, as it enables students to understand and follow the reasoning behind the mathematical theory being taught. At the same time, it allows them to see and comprehend the role played by the different elements involved in proving new mathematical results, as well as the assumptions that need to be imposed. Naturally, the course aims to foster a critical attitude toward any mathematical statement, as well as the intuition necessary to assess the appropriateness of various mathematical models to a wide range of real-world situations (physical, biological, economic, etc.), through applied problems where modeling plays a central role.

In the problem-solving classes, practical problems will be addressed. Special attention will also be paid to the students' oral and written communication. Furthermore, in the seminar and lab sessions, students will work-under the guidance of the instructor-on practical situations related to the content covered in the theory classes. These sessions will also allow both the instructor and the student to monitor the progress made in understanding the concepts and methods introduced in the theoretical component of the course.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First midterm exam	40%	4	0.16	KM10
Lab reports	20%	6	0.24	KM10
Retake exam	80%	4	0.16	KM10
Second midterm exam	40%	4	0.16	KM10

Continuous assessment:

Two midterm exams will be held: the first halfway through the course and the second at the end. Each exam will account for 40% of the final grade. Students must submit a report for each of the three laboratory sessions. These reports will collectively account for 20% of the final grade.

Single assessment:

Students opting for the single assessment will take a final exam covering theory and problem-solving, which will account for 80% of the final grade, and a practical exam on the same day, which will account for the remaining 20%.

Bibliography

Mendenhall, W., Scheaffer, R.L., Wackerly, D.D. Estadística matemática con aplicaciones. Grupo Editorial Iberoamérica (1986)

DeGroot, M.H. Probabilidad y estadística. Addison-Wesley Iberoamericana (1986)

Martín, F.J., Ruiz-Maya, L. Estadística I: Probabilidad. Editorial AC (1995)

Cuadras, C.M. Problemas de probabilidades y estadística. Vol 1: Probabilidades. EUB (1995)

Software

- R Core Team (2021). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.
- wxMaxima (C) 2004-2018 Andrej Vodopivec

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this <u>link</u>. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(PAUL) Classroom practices	1	Catalan	first semester	afternoon
(PLAB) Practical laboratories	1	Catalan	first semester	afternoon
(PLAB) Practical laboratories	2	Catalan	first semester	afternoon
(TE) Theory	1	Catalan	first semester	afternoon